2.1 The Phenomenon of Decoherence

2.1.1 Superpositions

The superposition principle forms the most fundamental kinematical concept of quantum theory. Its universality seems to have first been postulated by Dirac as part of the definition of his "ket-vectors", which he proposed as a complete[1] and general concept to characterize quantum states regardless of any *basis* of representation. They were later recognized by von Neumann as forming an abstract Hilbert space. The inner product (also needed to define a Hilbert space, and formally indicated by the distinction between "bra" and "ket" vectors) is not part of the kinematics proper, but required for the probability interpretation, which may be regarded as dynamics (as will be discussed). The third Hilbert space axiom (closure with respect to Cauchy series) is merely mathematically convenient, since one can never decide empirically whether the number of linearly independent physical states is infinite in reality, or just very large.

According to this kinematical superposition principle, *any* two physical states, $|1\rangle$ and $|2\rangle$, whatever their meaning, can be superposed in the form $c_1|1\rangle + c_2|2\rangle$, with complex numbers c_1 and c_2, to form a *new physical state* (to be be distinguished from a *state of information*). By induction, the principle can be applied to more than two, and even an infinite number, of states, and appropriately generalized to apply to a continuum of states. After postulating the linear Schrödinger equation in a general form, one may furthermore conclude that the superposition of two (or more) of its *solutions* forms again a solution. This is the *dynamical* version of the superposition principle.

Let me emphasize that this superposition pinciple is in drastic contrast to the concept of the "quantum" that gave the theory its name. Superpositions obeying the Schrödinger equation describe a deterministically evolving continuum rather than discrete quanta and stochastic quantum jumps. According to the theory of decoherence, these *effective* concepts "emerge" as a consequence of the superposition principle when universally and consistently applied.

A dynamical superposition principle (though in general with respect to real numbers only) is also known from classical waves which obey a linear wave equation. Its validity is then restricted to cases where these equations apply, while the quantum superposition principle is meant to be universal and exact. However, while the physical meaning of classical superpositions is usually obvious, that of quantum mechanical superpositions has to be somehow determined. For example, the interpretation of a superposition $\int dq\, e^{ipq}\, |q\rangle$ as representing a state of momentum p can be derived from "quantization rules", valid for systems whose classical counterparts are known in their Hamiltonian form (see Sect. 2.2). In other cases, an interpretation may be derived from the dynamics or has to be based on experiments.

Dirac emphasized another (in his opinion even more important) difference: all non-vanishing components of (or projections from) a superposition are "in some sense contained" in it. This formulation seems to refer to an *ensemble* of physical states, which would imply that their description by formal "quantum states" is *not* complete. Another interpretation asserts that it is the (Schrödinger) *dynamics* rather than the concept of quantum states which is incomplete. States found in measurements would then have to *arise* from an initial state by means of an indeterministic "collapse of the wave function". Both interpretations meet serious difficulties when consistently applied (see Sect. 2.3).

In the third edition of his textbook, Dirac (1947) starts to explain the superposition principle by discussing one-particle states, which can be described by Schrödinger waves in three-dimensional space. This is an important application, although its similarity with classical waves may also be misleading. Wave functions derived from the quantization rules are defined on their classical configuration space, which happens to coincide with normal space only for a single mass point. Except for this limitation, the two-slit interference experiment, for example, (effectively a two-state superposition) is known to be very instructive. Dirac's second example, the superposition of two basic photon polarizations, no longer corresponds to a spatial wave. These two basic states "contain" all possible photon polarizations. The electron spin, another two-state system, exhausts the group SU(2) by a two-valued representation of spatial rotations, and it can be studied (with atoms or neutrons) by means of many variations of the Stern–Gerlach experiment. In his lecture notes (Feynman, Leighton, and Sands 1965), Feynman describes the maser mode of the ammonia molecule as another (very different) two-state system.

All these examples make essential use of superpositions of the kind $|\alpha\rangle = c_1|1\rangle + c_2|2\rangle$, where the states $|1\rangle$, $|2\rangle$, and (all) $|\alpha\rangle$ can be observed as *physically different* states, and distinguished from one another in an appropriate setting. In the two-slit experiment, the states $|1\rangle$ and $|2\rangle$ represent the partial Schrödinger waves that pass through one or the other slit. Schrödinger's wave function can itself be understood as a consequence of the superposition principle by being viewed as the amplitudes $\psi_\alpha(q)$ in the superposition of "classical" configurations q (now represented by corresponding quantum states $|q\rangle$ or their narrow wave packets). In this case of a system with a known classical counterpart, the superpositions $|\alpha\rangle = \int dq\, \psi_\alpha(q)|q\rangle$ are assumed to define all quantum states. They may represent new observable properties (such as energy or angular momentum), which are not simply functions of the configuration, $f(q)$, only as a nonlocal *whole*, but not as an integral over corresponding local densities (neither on space nor on configuration space).

Since Schrödinger's wave function is thus defined on (in general high-dimensional) configuration space, increasing its amplitude does not describe an increase of intensity or energy density, as it would for classical waves in three-dimensional space. Superpositions of the intuitive product states of composite quantum systems may not only describe particle exchange symmetries (for bosons and fermions); in the general case they lead to the fundamental concept of *quantum nonlocality*. The latter has to be distinguished from a mere *extension* in space (characterizing extended classical objects). For example, molecules in energy eigenstates are incompatible with their atoms being in definite quantum states themselves. Although the importance of this "entanglement" for many observable quantities (such as the binding energy of the helium atom, or total angular momentum) had been well known, its consequence of violating Bell's inequalities (Bell 1964) seems to have surprised many physicists, since this result strictly excluded all local theories conceivably underlying quantum theory. However, quantum nonlocality appears paradoxical only when one attempts to interpret the wave function in terms of an ensemble of local properties, such as "particles". If reality were *defined* to be local ("in space and time"), then it would indeed conflict with the empirical actuality of a general superposition. Within the quantum formalism, entanglement also leads to decoherence, and in this way it *explains* the classical appearance of the observed world in quantum mechanical terms. The application of this program is the main subject of this book (see also Zurek 1991, Mensky 2000, Tegmark and Wheeler 2001, Zurek 2001, or www.decoherence.de).

The predictive power of the superposition principle became particularly evident when it was applied in an ingenious step to postulate the existence of superpositions of states with different particle numbers (Jordan and Klein 1927). Their meaning is illustrated, for example, by "coherent states" of different photon numbers, which may represent quasi-classical states of the electromagnetic field (cf. Glauber 1963). Such dynamically arising (and in many cases experimentally confirmed) superpositions are often misinterpreted as representing "virtual" states, or mere probability amplitudes for the occurrence of "real" states that are assumed to possess definite particle number. This would be as mistaken as replacing a hydrogen wave function by the probability distribution $p(\mathbf{r}) = |\psi(\mathbf{r})|^2$, or an entangled state by an *ensemble* of product states (or a two-point function). A superposition is in general observably different from an ensemble consisting of its components with corresponding probabilities.

Another spectacular success of the superposition principle was the prediction of new particles formed as superpositions of K-mesons and their antiparticles (Gell-Mann and Pais 1955, Lee and Yang 1956). A similar model describes the recently confirmed "neutrino oscillations" (Wolfenstein 1978), which are superpositions of energy eigenstates.

The superposition principle can also be successfully applied to states that may be generated by means of symmetry transformations from asymmetric ones. In classical mechanics, a symmetric Hamiltonian means that each asymmetric *solution* (such as an elliptical Kepler orbit) implies other solutions, obtained by applying the symmetry transformations (e.g. rotations). Quantum theory requires *in addition* that all their superpositions also form solutions (cf. Wigner 1964, or Gross 1995; see also Sect. 9.6). A complete set of energy eigenstates can then be constructed by means of *irreducible linear representations* of the dynamical symmetry group. Among them are usually symmetric ones (such as s-waves for scalar particles) that need not have a counterpart in classical mechanics.

A great number of novel applications of the superpositon principle have been studied experimentally or theoretically during recent years. For example, superpositions of different "classical" states of laser modes ("mesoscopic Schrödinger cats") have been prepared (Monroe *et al.* 1996), the entanglement of photon pairs has been confirmed to persist over tens of kilometers (Tittel *et al.* 1998), and interference experiments with fullerene molecules were successfully performed (Arndt *et al.* 1999). Even superpositions of a macroscopic current running in opposite directions have been shown to exist, and confirmed to be different from a state with *two* (cancelling) currents (Mooij *et al.* 1999, Friedman *et al.* 2000). Quantum computers, now under intense investigation, would have to perform "parallel" (but not *spatially* separated) calculations, while forming one superposition that may later have a coherent effect. So-called quantum teleportation requires the *advanced preparation* of an entangled state of distant systems (cf. Busch et al. 2001 for a consistent description in quantum mechanical terms). One of its components may then later be selected by a *local* measurement in order to determine the state of the other (distant) system.

Whenever an experiment was technically feasible, all components of a superposition have been shown to *act* coherently, thus proving that they *exist* simultaneously. It is surprising that many physicists still seem to regard superpositions as representing some state of ignorance (merely characterizing unpredictable "events"). After the fullerene experiments there remains but a minor step to discuss conceivable (though hardly realizable) interference experiments with a conscious observer. Would he have one or many "minds" (when being aware of his path through the slits)?

The most general quantum states seem to be superpositions of different classical fields on three- or higher-dimensional space.[2] In a perturbation expansion in terms of free "particles" (wave modes) this leads to terms corresponding to Feynman diagrams, as shown long ago by Dyson (1949). The path integral describes a *superposition* of paths, that is, the propagation of wave functionals according to a generalized Schrödinger equation, while the individual paths under the integral have no *physical* meaning by themselves. (A similar method could be used to describe the propagation of classical waves.) Wave functions will here always be understood in the generalized sense of *wave functionals* if required.

One has to keep in mind this universality of the superposition principle and its consequences for *individually* observable physical properties in order to appreciate the meaning of the program of decoherence. Since quantum coherence is far more than the appearance of spatial interference fringes observed statistically in series of "events", decoherence must not simply be understood in a classical sense as their washing out under *fluctuating* environmental conditions.

2.1.2 Superselection Rules

In spite of this success of the superposition principle it soon became evident that not *all* conceivable superpositions are found in Nature. This led some physicists to postulate "superselection rules", which restrict this principle by axiomatically excluding certain superpositions (Wick, Wightman,

[2] The empirically correct "pre-quantum" configurations for fermions are given by spinor fields on space, while the apparently observed particles are no more than the consequence of decoherence by means of *local* interactions with the environment (see Chap. 3). Field amplitudes (such as $\psi(\mathbf{r})$) seem to form the general arguments of the wave function(al) Ψ, while space points $\mathbf{r}$ appear as their "indices" – not as dynamical position variables. Neither a "second quantization" nor a wave-particle dualism are required. N-particle wave functions may be obtained as a non-relativistic approximation by applying the superposition principle (as a "quantization procedure") to these apparent particles instead of the correct pre-quantum variables (fields), which are not directly observable for fermions. The concept of particle permutations then becomes a *redundancy* (see Sect. 9.6). Unified field theories are usually expected to provide a general (supersymmetric) pre-quantum field and its Hamiltonian.

and Wigner 1970, Streater and Wightman 1964). There are also attempts to *derive* some of these superselection rules from other principles, which can be postulated in quantum field theory (see Chaps. 6 and 7). In general, these principles merely exclude "unwanted" consequences of a general superposition principle by hand.

Most disturbing in this sense seem to be superpositions of states with integer and half-integer spin (bosons and fermions). They violate invariance under 2π-rotations (see Sect. 6.2), but such a non-invariance has been experimentally confirmed in a different way (Rauch *et al.* 1975). The theory of supersymmetry (Wess and Zumino 1971) *postulates* superpositions of bosons and fermions. Another supposedly "fundamental" superselection rule forbids superpositions of different charge. For example, superpositions of a proton and a neutron have never been directly observed, although they occur in the *isotopic spin* formalism. This (dynamically broken) symmetry was later successfully generalized to SU(3) and other groups in order to characterize further intrinsic degrees of freedom. However, superpositions of a proton and a neutron may "exist" within nuclei, where isospin-dependent self-consistent potentials may arise from an *intrinsic symmetry breaking*. Similarly, superpositions of different charge are used to form BCS states (Bardeen, Cooper, and Schrieffer 1957), which describe the intrinsic properties of superconductors. In these cases, definite charge values have to be projected out (see Sect. 9.6) in order to describe the observed physical objects, which do obey the charge superselection rule.

Other limitations of the superposition principle are less clearly defined. While elementary particles are described by means of wave functions (that is, superpositions of different positions or other properties), the moon seems always to be at a definite place, and a cat is either dead or alive. A general superposition principle would even allow superpositions of a cat and a dog (as suggested by Joos). They would have to define a "new animal" – analogous to a K_{long}, which is a superposition of a K-meson and its antiparticle. In the Copenhagen interpretation, this difference is attributed to a strict conceptual separation between the microscopic and the macroscopic world. However, where is the border line that distinguishes an n-particle state of quantum mechanics from an N-particle state that is classical? Where, precisely, does the superposition principle break down?

Chemists do indeed know that a border line seems to exist deep in the microscopic world (Primas 1981, Woolley 1986). For example, most molecules (save the smallest ones) are found with their nuclei in definite (usually rotating and/or vibrating) classical "configurations", but hardly ever in superpositions thereof, as it would be required for energy or angular momentum eigenstates. The latter are observed for hydrogen and other *small* molecules. Even chiral states of a sugar molecule appear "classical", in contrast to its parity and energy eigenstates, which correctly describe the otherwise analogous maser mode states of the ammonia molecule (see Sect. 3.2.4 for details).

Does this difference mean that quantum mechanics breaks down already for very small particle number?

Certainly not in general, since there are well established superpositions of many-particle states: phonons in solids, superfluids, SQUIDs, white dwarf stars and many more! All properties of macroscopic bodies which can be calculated quantitatively are consistent with quantum mechanics, but not with any microscopic classical description. As will be demonstrated throughout the book, the theory of decoherence is able to *explain* the apparent differences between the quantum and the classical world under the assumption of a *universally valid* quantum theory.

The attempt to derive the absence of certain superpositions from (exact or approximate) conservation laws, which forbid or suppress transitions between their corresponding components, would be insufficient. This "traditional" explanation (which seems to be the origin of the name "superselection rule") was used, for example, by Hund (1927) in his arguments in favor of the chiral states of molecules. However, small or vanishing transition rates require *in addition* that superpositions were absent initially for all these molecules (or their constituents from which they formed). Similarly, charge conservation does *not* explain the charge superselection rule! Negligible wave packet dispersion (valid for large mass) may prevent initially presumed wave packets from growing wider, but this initial condition is quantitatively insufficient to explain the quasi-classical appearance of mesoscopic objects, such as small dust grains or large molecules (see Sect. 3.2.1), or even that of celestial bodies in chaotic motion (Zurek and Paz 1994). Even initial conditions for conserved quantities would in general allow one only to exclude *global* superpositions, but not local ones (Giulini, Kiefer and Zeh 1995).

So how can superselection rules be explained within quantum theory?

2.1.3 Decoherence by "Measurements"

Other experiments with quantum objects have taught us that interference, for example between partial waves, disappears when the property characterizing these partial waves is *measured*. Such partial waves may describe the passage through different slits of an interference device, or the two beams of a Stern–Gerlach device ("*Welcher Weg* experiments"). This loss of coherence is indeed required by mere logic once measurements are assumed to lead to definite results. In this case, the frequencies of events on the detection screen measured in coincidence with a *certain* passage can be counted separately, and thus have to be added to define the total probabilities.[3] It is therefore a *plausible*

[3] Mere logic does *not* require, however, that the frequencies of events on the screen which follow the observed passage through slit 1 of a two-slit experiment, say, are the same as those without measurement, but with slit 2 closed. This distinction would be relevant in Bohm's theory (Bohm 1952) if it allowed nondisturbing measurements of the (now *assumed*) passage through one definite slit (as it does *not* in

experience that the interference disappears also when the passage is "measured" without registration of a definite result. The latter may be *assumed* to have become a "classical fact" as soon the measurement has irreversibly "occurred". A quantum phenomenon may thus "become a phenomenon" *without* being observed (in contrast to this early formulation of Bohr's, which is in accordance with Heisenberg's idealistic statement about a trajectory coming into being by its observation – while Bohr later spoke of objective irreversible events occurring in the counter). However, what presicely is an irreversible quantum event? According to Bohr, it can *not* be dynamically analyzed.

Analysis within the quantum mechanical formalism demonstrates nonetheless that the essential condition for this "decoherence" is that complete information about the passage is carried away in some *physical* form (Zeh 1970, 1973, Mensky 1979, Zurek 1981, Caldeira and Leggett 1983, Joos and Zeh 1985). Possessing "information" here means that the physical state of the environment is now uniquely *quantum correlated* (entangled) with the relevant property of the system (such as a passage through a specific slit). This need *not* happen in a controllable form (as in a measurement): the "information" may as well be created in the form of noise. However, in contrast to statistical correlations, quantum correlations define *pure* (completly defined) nonlocal states, and thus *individual physical* properties, such as the total spin of spatially separated objects. Therefore, one cannot *explain* entanglement in terms of the concept of information (cf. Brukner and Zeilinger 2000). This terminology would mislead to the popular misunderstanding of the collapse as a "mere increase of information" (which would require an initial ensemble describing ignorance). Since environmental decoherence affects individual physical states, it can *neither* be the consequence of phase averaging in an ensemble, *nor* one of phases fluctuating uncontrollably in time (as claimed in some textbooks). For example, nonlocal entanglement exists in the *static* quantum state of a relativstic physical vacuum (even though it is then often visualized in terms of particles as "vacuum fluctuations").

When is unambiguous "information" carried away? If a macroscopic object had the opportunity of passing through two slits, we would always be able to convince ourselves of its choice of a path by simply opening our eyes in order to "look". This means that in this case there is plenty of light that contains information about the path (even in a controllable manner that allows "looking"). Interference between different paths never occurs, since the

order to remain indistinguishable from quantum theory). The fact that these two quite different situations (closing slit 2 or measuring the passage through slit 1) lead to exactly the same subsequent frequencies, which differ entirely from those that are *defined* by this theory when not measured or selected, emphasizes its extremely artificial nature (see also Englert *et al.* 1992, or Zeh 1999). The predictions of quantum theory are here simply reproduced by leaving the Schrödinger equation unaffected and universally valid, identical with Everett's assumptions (Everett 1957). In both these theories the wave function is (for good reasons) regarded as a *real* physical object (cf. Bell 1981).

path is evidently "continuously measured" by light. The common textbook argument that the interference pattern of macroscopic objects be too fine to be observable is entirely irrelevant. However, would it then not be sufficient to dim the light in order to reproduce (in principle) a quantum mechanical interference pattern for macroscopic objects?

This could be investigated by means of more sophisticated experiments with mesoscopic objects (see Brune *et al.* 1996). However, in order to precisely determine the subtle limit where measurement by the environment becomes negligible, it is more economic first to apply the established theory which is known to describe such experiments. Thereby we have to take into account the quantum nature of the environment, as discussed long ago by Brillouin (1962) for an information medium in general. This can usually be done easily, since the quantum theory of interacting systems, such as the quantum theory of particle scattering, is well understood. Its application to decoherence requires that one averages over all unobserved degrees of freedom. In technical terms, one has to "trace out the environment" after it has interacted with the considered system. This procedure leads to a quantitative theory of decoherence (cf. Joos and Zeh 1985). Taking the trace is based on the probability interpretation applied to the environment (averaging over all possible outcomes of measurements), even though this environment is *not* measured. (The precise physical meaning of these formal concepts will be discussed in Sect. 2.4.)

Is it possible to explain *all* superselection rules in this way as an effect induced by the environment[4] – including the existence and position of the border line between microscopic and macroscopic behaviour in the realm of molecules? This would mean that the universality of the superposition principle could be maintained – as is indeed the basic idea of the *program of decoherence* (Zeh 1970, Zurek 1982; see also Chap. 4 of Zeh 2001). If physical states are thus exclusively described by wave functions rather than by points in configuration space – as originally intended by Schrödinger *in space* by means of narrow wave packets instead of particles – then no uncertainty relations are available *for states* in order to explain the probabilistic aspects of quantum theory: the Fourier theorem applies to a *given* wave function(al).

As another example, consider two states of different charge. They interact very differently with the electromagnetic field even in the absence of radiation: their Coulomb fields carry complete "information" about the total charge *at any distance*. The quantum state of this field would thus decohere a superposition of different charges if considered as a quantum system in a *bounded* region of space (Giulini, Kiefer, and Zeh 1995). This instantaneous action of decoherence at an arbitrary distance by means of the Coulomb field gives it the appearance of a kinematical effect, although it is based on the

[4] It would be sufficient, for this purpose, to use an *internal* "environment" (unobserved degrees of freedom), but the assumption of a closed system would in general be unrealistic.

dynamical law of charge conservation, compatible with a *retarded* field that would "measure" the charge (see Sect. 6.4).

There are many other cases where the unavoidable effect of decoherence can easily be imagined without any calculation. For example, superpositions of macroscopically different electromagnetic fields, $f(\mathbf{r})$, may be described by a field functional $\Psi[f(\mathbf{r})]$. However, any charged particle in a sufficiently narrow wave packet would then evolve into different packets, depending on the field f, and thus become entangled with the state of the quantum field (Kübler and Zeh 1973, Kiefer 1992, Zurek, Habib, and Paz 1993; see also Sect. 4.1.2). The particle can be said to "measure" the quantum state of the field. Since charged particles are in general abundant in the environment, no superpositions of macroscopically different electromagnetic fields (or different "mean fields" in other cases) are observed under normal conditions. This result is related to the difficulty of preparing and maintaining "squeezed states" of light (Yuen 1976) – see Sect. 3.3.3.1. Therefore, the field appears to *be* in one of its classical states (Sect. 4.1.2).

In all these cases, this conclusion requires that the quasi-classical states (or "pointer states" in measurements) are robust (dynamically stable) under natural decoherence, as pointed out already in the first paper on decoherence (Zeh 1970; see also Diósi and Kiefer 2000).

A particularly important example of a quasiclassical field is the metric of general relativity (with classical *states* described by spatial geometries on space-like hypersurfaces – see Sect. 4.2). Decoherence caused by all kinds of matter can therefore explain the absence of superpositions of macroscopically distinct spatial curvatures (Joos 1986, Zeh 1986, 1988, Kiefer 1987), while *microscopic* superpositions would describe those hardly ever observable gravitons.

Superselection rules thus arise as a straightforward consequence of quantum theory under realistic assumptions. They have nonetheless been discussed mainly in mathematical physics – apparently under the influence of von Neumann's and Wigner's "orthodox" interpretation of quantum mechanics (see Wightman 1995 for a review). Decoherence by "continuous measurement" seems to form the most fundamental irreversible process in Nature. It applies even where thermodynamical concepts do *not* (such as for individual molecules – see Sect. 3.2.4), or when any exchange of heat is entirely negligible. Its time arrow of "microscopic causality" requires a Sommerfeld radiation condition for microscopic scattering (similar to Boltzmann's chaos), *viz.*, the absence of any dynamically relevant *initial* correlations, which would define a "conspiracy" in common terminology (Joos and Zeh 1985, Zeh 2001).

2.2 Observables as a Derived Concept

Measurements are usually described by means of "observables", formally represented by Hermitian operators, and introduced in addition to the concepts of quantum states and their dynamics as a fundamental and independent ingredient of quantum theory. However, even though often forming the starting point of a formal quantization procedure, this ingredient should not be separately required if physical states are well described by these formal quantum states. This understanding, to be further explained below, complies with John Bell's quest for the replacement of observables with "beables" (see Bell 1987). It was for this reason that his preference shifted from Bohm's theory to collapse models (where wave functions are assumed to completely describe *reality*) during his last years.

Let $|\alpha\rangle$ be an arbitrary quantum state, defined operationally (up to a complex numerical factor) by a "complete preparation" procedure. The phenomenological probability for finding the system during an appropriate measurement in another quantum state $|n\rangle$, say, is given by means of their inner product as $p_n = |\langle n \mid \alpha\rangle|^2$ (where both states are assumed to be normalized). The state $|n\rangle$ is here defined by the specific measurement. (In a position measurement, for example, the number n has to be replaced with the continuous coordinates x, y, z, leading to the "improper" Hilbert states $|\mathbf{r}\rangle$.) For measurements of the "first kind" (to which all others can be approximately reduced – see Sect. 2.3), the system will again be found in the state $|n\rangle$ with certainty if the measurement is immediately repeated. *Preparations* can be regarded as such measurements which *select* a certain subset of outcomes for further measurements. n-preparations are therefore also called n-filters, since all "not-n" results are thereby excluded from the subsequent experiment proper. The above probabilities can also be written in the form $p_n = \langle\alpha \mid P_n \mid \alpha\rangle$, with an "observable" $P_n := |n\rangle\langle n|$, which is thus *derived* from the kinematical concept of quantum *states*.

Instead of these special "n or not-n measurements" (with fixed n), one can also perform more general "n_1 or n_2 or ... measurements", with all n_i's mutually exclusive ($\langle n_i|n_j\rangle = \delta_{ij}$). If the states forming such a set $\{|n\rangle\}$ are pure and exhaustive (that is, complete, $\sum P_n = \mathbb{1}$), they represent a basis of the corresponding Hilbert space. By introducing an arbitrary "measurement scale" a_n, one may construct *general* observables $A = \sum |n\rangle a_n \langle n|$, which permit the definition of "expectation values" $\langle\alpha \mid A \mid \alpha\rangle = \sum p_n a_n$. In the special case of a yes-no measurement, one has $a_n = \delta_{nn_0}$, and expectation values become probabilities. Finding the state $|n\rangle$ during a measurement is then also expressed as "finding the value a_n of an observable". A change of scale, $b_n = f(a_n)$, describes the *same* physical measurement; for position measurements of a particle it would simply represent a coordinate transformation. Even a measurement of the particle's potential energy is equivalent to a position measurement (up to degeneracy) if the function $V(\mathbf{r})$ is *given*.

According to this definition, quantum expectation values must not be understood as mean values in an ensemble that represents ignorance of the precise state. Rather, they have to be interpreted as probabilities for *potentially arising* quantum states $|n\rangle$ – regardless of the latters' interpretation. If the set $\{|n\rangle\}$ of such potential states forms a basis, any state $|\alpha\rangle$ can be represented as a superposition $|\alpha\rangle = \sum c_n|n\rangle$. In general, it neither forms an n_0-state nor any not-n_0 state. Its dependence on the complex coefficients c_n requires that states which differ from one another by a numerical factor must be different "in reality". This is true even though they represent the same "ray" in Hilbert space and cannot, according to the measurement postulate, be distinguished operationally. The states $|n_1\rangle + |n_2\rangle$ and $|n_1\rangle - |n_2\rangle$ could not be physically different from another if $|n_2\rangle$ and $-|n_2\rangle$ were the *same* state. (Only a *global* numerical factor would be "redundant".) For this reason, projection operators $|n\rangle\langle n|$ are insufficient to characterize quantum states (cf. also Mirman 1970).

The expansion coefficients c_n, relating physically meaningful states – for example those describing different spin directions or different versions of the K-meson – must in principle be determined (relative to one another) by appropriate experiments. However, they can often be derived from a previously known (or conjectured) classical theory by means of "quantization rules". In this case, the classical configurations q (such as particle positions or field variables) are *postulated* to parametrize a basis in Hilbert space, $\{|q\rangle\}$, while the canonical momenta p parametrize another one, $\{|p\rangle\}$. Their corresponding observables, $Q = \int dq\, |q\rangle q\langle q|$ and $P = \int dp\, |p\rangle p\langle p|$, are required to obey commutation relations in analogy to the classical Poisson brackets. In this way, they form an important *tool* for constructing and interpreting the specific Hilbert space of quantum states. These commutators essentially determine the unitary transformation $\langle p \mid q\rangle$ (e.g. as a Fourier transform $\mathrm{e}^{\mathrm{i}pq}$) – thus more than what could be defined by means of the projection operators $|q\rangle\langle q|$ and $|p\rangle\langle p|$. This algebraic procedure is mathematically very elegant and appealing, since the Poisson brackets and commutators may represent generalized symmetry transformations. However, the *concept* of observables (which form the algebra) can be derived from the more fundamental one of state vectors and their inner products, as described above.

Physical states are assumed to vary in time in accordance with a dynamical law – in quantum mechanics of the form $\mathrm{i}\partial_t|\alpha\rangle = H|\alpha\rangle$. In contrast, a measurement device is usually defined regardless of time. This must then also hold for the observable representing it, or for its eigenbasis $\{|n\rangle\}$. The probabilities $p_n(t) = |\langle n \mid \alpha(t)\rangle|^2$ will therefore vary with time according to the time-dependence of the physical states $|\alpha\rangle$. It is well known that this (Schrödinger) time dependence is formally equivalent to the (inverse) time dependence of observables (or the reference states $|n\rangle$). Since observables "correspond" to classical *variables*, this time dependence appeared suggestive in the Heisenberg–Born–Jordan algebraic approach to quantum theory.

However, the absence of *dynamical states* $|\alpha(t)\rangle$ from this Heisenberg picture, a consequence of insisting on *classical* kinematical concepts, leads to paradoxes and conceptual inconsistencies (complementarity, dualism, quantum logic, quantum information, and all that).

An environment-induced superselection rule means that certain superpositions are highly unstable with respect to decoherence. It is then impossible in practice to construct measurement devices for them. This *empirical* situation has led some physicists to *deny the existence* of these superpositions and their corresponding observables – either by postulate or by formal manipulations of dubious interpretation, often including infinities. In an attempt to circumvent the measurement problem (that will be discussed in the following section), they often simply *regard* such superpositions as "mixtures" once they have formed according to the Schrödinger equation (cf. Primas 1990).

While *any* basis $\{|n\rangle\}$ in Hilbert space defines formal probabilities, $p_n = |\langle n|\alpha\rangle|^2$, only a basis consisting of states that are not immediately destroyed by decoherence defines a practically "realizable observable". Since realizable observables usually form a genuine subset of *all* formal observables (diagonalizable operators), they must contain a nontrivial "center" in algebraic terms. It consists of those of them which commute with all the rest. Observables forming the center may be regarded as "classical", since they can be measured simultaneously with *all* realizable ones. In the algebraic approach to quantum theory, this center appears as part of its axiomatic structure (Jauch 1968). However, since the condition of decoherence has to be considered quantitatively (and may even vary to some extent with the specific nature of the environment), this algebraic classification remains an approximate and dynamically emerging scheme.

These "classical" observables thus define the subspaces into which superpositions decohere. Hence, even if the superposition of a right-handed and a left-handed chiral molecule, say, *could* be prepared by means of an appropriate (very fast) measurement of the first kind, it would be destroyed before the measurement may be repeated for a test. In contrast, the chiral states of all individual molecules in a bag of sugar are "robust" in a normal environment, and thus retain this property *individually* over time intervals which by far exceed thermal relaxation times. This stability may even be increased by the quantum Zeno effect (Sect. 3.3.1). Therefore, chirality appears not only classical, but also as an approximate constant of the motion that has to be taken into account in the definition of thermodynamical ensembles (see Sect. 2.3).

The above-used description of measurements of the first kind by means of probabilities for transitions $|\alpha\rangle \to |n\rangle$ (or, for that matter, by corresponding observables) is phenomenological. However, measurements should be described *dynamically* as interactions between the measured system and the measurement device. The observable (that is, the measurement basis) should thus be derived from the corresponding interaction Hamiltonian and the initial state of the device. As discussed by von Neumann (1932), this interaction must be diagonal with respect to the measurement basis (see also Zurek 1981). Its diagonal matrix elements are operators which act on the quantum state of the device in such a way that the "pointer" moves into a position appropriate for being read, $|n\rangle|\Phi_0\rangle \rightarrow |n\rangle|\Phi_n\rangle$. Here, the first ket refers to the system, the second one to the device. The states $|\Phi_n\rangle$, representing different pointer positions, must approximately be mutually orthogonal, and "classical" in the explained sense.

Because of the dynamical superposition principle, an initial superposition $\sum c_n|n\rangle$ does *not* lead to definite pointer positions (with their empirically observed frequencies). If decoherence is neglected, one obtains their *entangled superposition* $\sum c_n|n\rangle|\Phi_n\rangle$, that is, a state that is different from all potential measurement outcomes $|n\rangle|\Phi_n\rangle$. This dilemma represents the "quantum measurement problem" to be discussed in Sect. 2.3. Von Neumann's interaction is nonetheless regarded as the first step of a measurement (a "premeasurement"). Yet, a collapse seems still to be required – now in the measurement device rather than in the microscopic system. Because of the entanglement between system and apparatus, it then affects the total system.[5]

If, in a certain measurement, a whole subset of states $|n\rangle$ leads to the same pointer position $|\Phi_{n_0}\rangle$, these states are not distinguished in this measurement. The pointer state $|\Phi_{n_0}\rangle$ now becomes dynamically correlated with the whole *projection* of the initial state, $\sum c_n|n\rangle$, on the subspace spanned by this subset. A corresponding *collapse* was indeed postulated by Lüders (1951) in his generalization of von Neumann's "first intervention" (Sect. 2.3).

In this dynamical sense, the interaction with an appropriate measuring device *defines* an observable up to arbitrary monotoneous scale transformations. The time dependence of observables according to the Heisenberg picture would thus describe an imaginary time dependence of the states of this device (its pointer states), paradoxically controlled by the intrinsic Hamiltonian of the *system*.

The question of whether a formal observable (that is, a diagonalizable operator) can be physically realized can only be answered by taking into account the unavoidable environment of the system (while the measurement device is *always* assumed to decohere into its macroscopic pointer states). However, environment-induced decoherence by itself does not solve the measurement problem, since the "pointer states" $|\Phi_n\rangle$ may be assumed to *include* the total environment (the "rest of the world"). Identifying the thus arising

[5] Some authors seem to have taken the phenomenological collapse in the *microscopic system* by itself too literally, and therefore disregarded the state of the measurement device in their measurement theory (see Machida and Namiki 1980, Srinivas 1984, and Sect. 9.1). Their approach is based on the assumption that quantum states must always exist for all systems. This would be in conflict with quantum nonlocality, even though it may be in accordance with early interpretations of the quantum formalism.

global superposition with an *ensemble* of states, represented by a statistical operator ρ, that merely leads to the same *expectation values* $\langle A \rangle = \mathrm{tr}(\mathrm{A}\rho)$ for a *limited* set of observables $\{A\}$ would obviously beg the question. This argument is nonetheless found wide-spread in the literature (cf. Haag 1992, who used the subset of all *local* observables).

In Sect. 2.4, statistical operators ρ will be *derived* from the concept of quantum states as a tool for calculating expectation values, while the latter are defined, as described above, by means of probabilities for the occurrence of new states in measurements. In the Heisenberg picture, ρ is often regarded as in some sense representing the *ensemble of potential "values"* for all observables that are here postulated to formally replace the classical variables. This interpretation is suggestive because of the (incomplete) formal analogy of ρ to a classical phase space distribution. However, the prospective "values" would be *physically* meaningful only if they characterized different physical *states* (such as pointer states). Note that Heisenberg's uncertainty relations refer to potential outcomes which may arise in different (mutually exclusive) measurements.

2.3 The Measurement Problem

The superposition of different measurement outcomes, resulting according to the Schrödinger equation (as discussed above), demonstrates that a "naive ensemble interpretation" of quantum mechanics in terms of incomplete knowledge is ruled out. It would mean that a quantum state (such as $\sum c_n|n\rangle|\Phi_n\rangle$) represents an ensemble of some as yet unspecified fundamental states, of which a subensemble (for example represented by the quantum state $|n\rangle|\Phi_n\rangle$) may be "picked out by a mere increase of information". If this were true, then the subensemble resulting from this measurement could in principle be traced back in time by means of the Schrödinger equation in order to determine also the initial state more completely (to "postselect" it – see Aharonov and Vaidman 1991 for an inappropriate attempt). In the above case this would lead to the initial quantum state $|n\rangle|\Phi_0\rangle$ that is *physically different* from – and thus inconsistent with – the superposition $(\sum c_n|n\rangle)|\Phi_0\rangle$ that had been prepared (whatever it *means*).

In spite of this simple argument, which demonstrates that an ensemble interpretation would require a complicated and miraculous nonlocal "background mechanism" in order to work consistently (cf. Footnote 3 regarding Bohm's theory), the ensemble interpretation of the wave function seems to remain the most popular one because of its pragmatic (though limited) value. A general and rigorous critical discussion of problems arising in an ensemble interpretation may be found in d'Espagnat's books (1976 and 1995).

A way out of this dilemma in terms of the wave function itself requires one of the following two possibilities: (1) a modification of the Schrödinger equation that explicitly describes a collapse (also called "spontaneous localization" – see Chap. 8), or (2) an Everett type interpretation, in which all measurement outcomes are assumed to coexist in one formal superposition, but to be *perceived* separately as a consequence of their dynamical decoupling under decoherence. While this latter suggestion may appear "extravagant" (as it requires myriads of coexisting parallel quasi-classical "worlds"), it is similar in principle to the conventional (though nontrivial) assumption, made tacitly in all classical descriptions of observations, that consciousness is *localized* in certain (semi-stable and suffiently complex) *spatial subsystems* of the world (such as human brains or parts thereof). For a dispute about which of the above-mentioned two possibilities should be preferred, the fact that environmental decoherence readily describes precisely the *apparently* occurring "quantum jumps" or "collapse events" (as will be discussed in great detail throughout this book) appears most essential.

The dynamical rules which are (explicitly or tacitly) used to describe the *effective* time dependence of quantum states thus represent a "dynamical dualism". This was first clearly formulated by von Neumann (1932), who distinguished between the unitary evolution according to the Schrödinger equation (remarkably his "zweiter Eingriff" or "second intervention"),

$$\mathrm{i}\hbar \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle \tag{2.1}$$

valid for isolated (absolutely closed) systems, and the "reduction" or "collapse of the wave function",

$$|\psi\rangle = \sum c_n |n\rangle \rightarrow |n_0\rangle \tag{2.2}$$

(remarkably his "*first* intervention"). The latter was to describe the stochastic transitions into the new state $|n_0\rangle$ during measurements. Their dynamical discontinuity had been anticipated by Bohr in the form of "quantum jumps" between his discrete electron *orbits*. Later, the *time-dependent* Schrödinger equation (2.1) for interacting systems was often regarded merely as a method of calculating probabilities for similar (individually unpredictable) discontinuous transitions between energy eigenstates (stationary *quantum* states) of atomic systems (Born 1926).[6] However, there does not seem to be any meaningful difference between quantum jumps into new states or subspaces and the "occurrence of values" for certain observables (cf. Sect. 2.2).

[6] Thus also Bohr (1928) in a subsection entitled "Quantum postulate and causality" about "the quantum theory": "...its essence may be expressed in the so-called quantum postulate, which attributes to any *atomic process* an essential discontinuity, or rather individuality, completely foreign to classical theories and symbolized by Planck's quantum of action" (my italics). The later revision of these early interpretations of quantum theory (required by the important role of entangled quantum states for much larger systems) seems to have gone unnoticed by many physicists.

In scattering theory, one usually probes only *part* of quantum mechanics by restricting consideration to asymptotic states and their probabilities (disregarding their superpositions). All quantum correlations between them then appear statistical ("classical"). Occasionally even the unitary scattering amplitudes $\langle m_{out}|n_{in}\rangle = \langle m| S |n\rangle$ are confused with the *probability* amplitudes $\langle\phi_m|\psi_n\rangle$ which describe measurements to find a state $|\phi_m\rangle$ in an initial $|\psi_n\rangle$. In his general S-matrix theory, Heisenberg temporarily speculated about deriving the latter from the former. Since macroscopic systems never become asymptotic because of their dynamical entanglment with the environment, they can not be described by an S-matrix at all.

The Born/von Neumann dynamical dualism was evidently the major motivation for an ignorance interpretation of the wave function, which attempts to explain the collapse *not* as a dynamical process in the system, but as an increase of *information* about it (the reduction of an ensemble of *possible* states). However, even though the dynamics of ensembles in classical description uses a formally similar dualism, an analogous interpretation in quantum theory leads to the severe (and apparently fatal) difficulties indicated above. They are often circumvented by the invention of "new rules of logic and statistics", which are *not* based on any ensemble interpretation or incomplete information.

If the state of a *classical* system is incompletely known, and the corresponding point p,q in phase space therefore replaced by an ensemble (a probability distribution) $\rho(p, q)$, this ensemble can be "reduced" by a new observation that leads to increased information. For this purpose, the system must interact in a controllable manner with the "observer" who holds the information (cf. Szilard 1929). His physical state of memory must thereby change in dependence on the property-to-be-measured of the observed system, leaving the latter unchanged in the ideal case (no "recoil"). According to *deterministic* dynamical laws, the ensemble entropy of the combined system, which initially contains the entropy corresponding to the unknown microscopic quantity, would remain constant if it were defined to include the entropy characterizing the final ensemble of different outcomes. Since the observer is assumed to "know" (to be aware of) his own state, this ensemble is reduced correspondingly, and the ensemble entropy defined *with respect to his state of information* is lowered.

This is depicted by the first step of Fig. 2.1, where ensembles of states are represented by areas. In contrast to many descriptions of Maxwell's demon, the observer (regarded as a device) is here subsumed into the ensemble description. *Physical entropy*, unlike ensemble entropy, is usually understood as a *local* (additive) concept, which neglects long range correlations for being "irrelevant", and thus approximately defines an *entropy density*. Physical and ensemble entropy are equal in the absence of correlations. The information I, given in the figure, measures the reduction of entropy according to the increased knowledge of the observer.

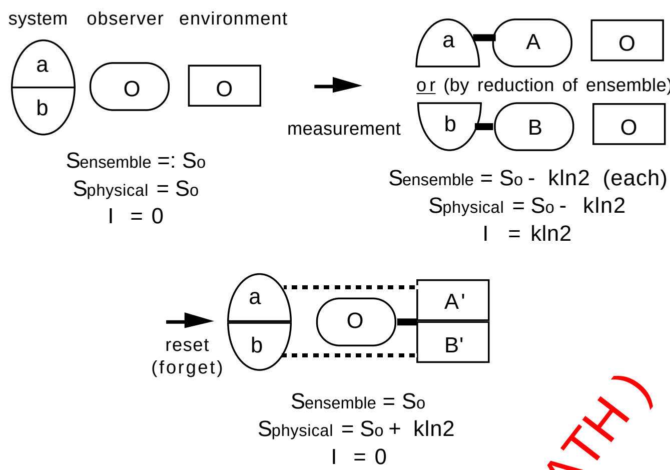

Fig. 2.1. Entropy relative to the state of information in an ideal *classical* measurement. Areas represent *sets* of microscopic states of the subsystems (while those of uncorrelated combined systems would be represented by their direct products). During the first step of the figure, the memory state of the observer changes deterministically from 0 to A or B, depending on the state a or b of the system to be measured. The second step depicts a subsequent reset, required if the measurement is to be repeated with the *same* device (Bennett 1973). A' and B' are effects which must thereby arise in the thermal environment in order to preserve the total ensemble entropy in accordance with presumed microscopic determinism. The "physical entropy" (*defined* to add for subsystems) measures the phase space of all microscopic degrees of freedom, including the property to be measured, while *depending on given* macroscopic variables. Because of its presumed additivity, this physical entropy neglects all remaining statistical correlations (dashed lines, which indicate *sums* of products of sets) for being "irrelevant" in the future – hence $S_{\text{physical}} \geq S_{\text{ensemble}}$. I is the amount of information held by the observer. The minimum initial entropy, S_0, is $k \ln 2$ in this simple case of two equally probable values a and b.

This description does not necessarily require a *conscious* observer (although it may ultimately rely upon him). It applies to any macroscopic measurement device, since physical entropy is not only defined to be local, but also relative to "given" macroscopic properties (as a function of them). The dynamical part of the measurement transforms "physical" entropy (here the ensemble entropy of the microscopic variables) deterministically into entropy of lacking information about controllable macroscopic properties. Before the observation is taken into account (that is, before the "or" is applied), both parts of the ensemble after the first step add up to give the ensemble entropy. When it *is* taken into account (as done by the numbers given in the figure), the ensemble entropy is reduced according to the information gained by the observer.

Any registration of information by the observer must use up his memory capacity ("blank paper"), which represents non-maximal entropy. If the same measurement is to be repeated, for example in a cyclic process that could be used to transform heat into mechanical energy (Szilard, *l.c.*), this capacity would either be exhausted at some time, or an equivalent amount of entropy must be absorbed by the environment (for example in the form of heat) in order to *reset* the measurement or registration device (second step of Fig. 2.1). The reason is that two *different* states cannot deterministically evolve into the same final state (Bennett 1973).[7] This argument is based on an arrow of time of "causality", which requires that all correlations possess *local causes* in their past (no "conspiracy"). The irreversible formation of "irrelevant" correlations then explains the increase of *physical* (local) entropy, while the ensemble entropy is conserved.

The unsurmountable problems encountered in an ensemble interpretation of the *wave function* (or of any other superposition, such as $|a\rangle + |b\rangle$) are reflected by the fact that there is no ensemble entropy that would represent the unknown property-to-be-measured (see the first step of Fig. 2.2 or 2.3 – cf. also Zurek 1984). The "ensemble entropy" is now *defined* by the "corresponding" expression $S_{\text{ensemble}} = -k\text{tr}\{\rho \ln \rho\}$ (but see Sect. 2.4 for the meaning of the density matrix ρ). If the entropy of observer plus environment is the same as in the classical case of Fig. 2.1, the total initial ensemble entropy is now lower; in the case of equal initial probabilities for a and b it is $S_0 - k \ln 2$. It would even vanish for pure states ϕ and χ of observer and environment, respectively: $(|a\rangle + |b\rangle)|\phi_0\rangle|\chi_0\rangle$. The Schrödinger evolution (depicted in Fig. 2.3) would then be described by three dynamical steps,

$$\begin{aligned}(|a\rangle + |b\rangle)\,|\phi_0\rangle\,|\chi_0\rangle &\to (|a\rangle\,|\phi_A\rangle + |b\rangle\,|\phi_B\rangle)\,|\chi_0\rangle \\ &\to |a\rangle\,|\phi_A\rangle\,|\chi_{A''}\rangle + |b\rangle\,|\phi_B\rangle\,|\chi_{B''}\rangle \\ &\to (|a\rangle\,|\chi_{A'A''}\rangle + |b\rangle\,|\chi_{B'B''}\rangle)\,|\phi_0\rangle \quad , \end{aligned} \tag{2.3}$$

with an "irrelevant" (inaccessible) final quantum correlation between system and environment as a relic from the initial superposition. In this unitary evolution, the two "branches" recombine to form a *nonlocal* superposition. It "exists, but it is not there". Its local unobservability characterizes an "apparent collapse" (as will be discussed). For a genuine collapse (Fig. 2.2), the final correlation would be statistical, and the ensemble entropy would increase, too.

As mentioned in Sect. 2.2, the general interaction dynamics that is required to describe "ideal" measurements according to the Schrödinger equation (2.1) is derived from the special case where the measured system is

[7] In *his* definitions, Bennett did not count the entropy of the microscopic ensemble a/b as physical entropy, because this variable is here assumed to be controllable, in contrast to the "thermal" (ergodic or uncontrollable) property A'/B'.

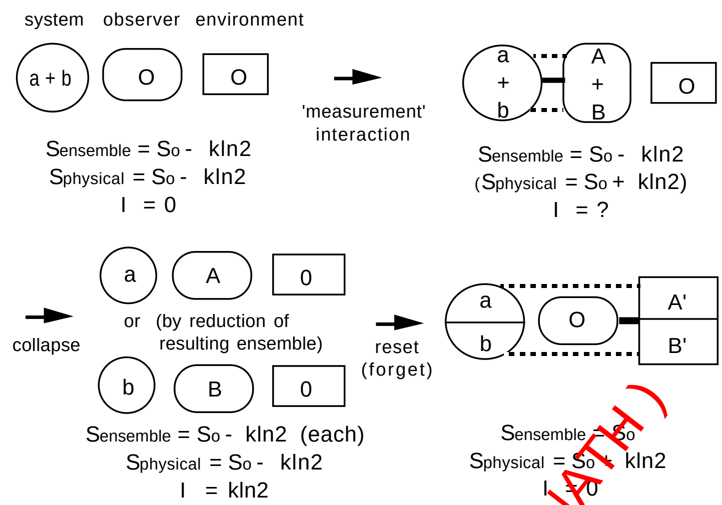

Fig. 2.2. Quantum measurement of a *superposition* $|a\rangle + |b\rangle$ by means of a *collapse* process, here assumed to be triggered by the macroscopic pointer position. The initial entropy is smaller by one bit than in Fig. 2.1 (and may in principle vanish), since there is no initial *ensemble* a/b for the property to be measured. Dashed lines *before* the collapse now represent quantum entanglement. (Compare the ensemble entropies with those of Fig. 2.1!) Increase of physical entropy in the first step is appropriate only if the arising entanglement is *regarded as irrelevant*. The collapse itself is often divided into *two* steps: first increasing the ensemble entropy by replacing the superposition with an ensemble, and then lowering it by reducing the ensemble (applying the "or" – for macroscopic pointers only). The increase of ensemble entropy, observed in the final state of the Figure, is a consequence of this first step of the collapse. It brings the entropy up to its classical initial value of Fig. 2.1

prepared in an eigenstate $|n\rangle$ before measurement (von Neumann 1932),

$$|n\rangle|\Phi_0\rangle \to |n\rangle|\Phi_n\rangle \quad . \tag{2.4}$$

Here, $|n\rangle$ corresponds to $|a\rangle$ or $|b\rangle$ in the figures, the pointer positions $|\Phi_n\rangle$ to the states $|\phi_A\rangle$ and $|\phi_B\rangle$. (During non-ideal measurements, the state $|n\rangle$ would change, too.) However, applied to an initial superposition, $\sum c_n|n\rangle$, the interaction according to (2.1) leads to an entangled superposition,

$$\left(\sum c_n|n\rangle\right)|\Phi_0\rangle \to \sum c_n|n\rangle|\Phi_n\rangle \quad . \tag{2.5}$$

As explained in Sect. 2.1.1, the resulting superposition represents an *individual physical state* that is different from all components appearing in this sum. While decoherence arguments teach us (see Chap. 3) that neglecting the environment of (2.5) is absolutely unrealistic if $|\Phi_n\rangle$ describes the pointer state of

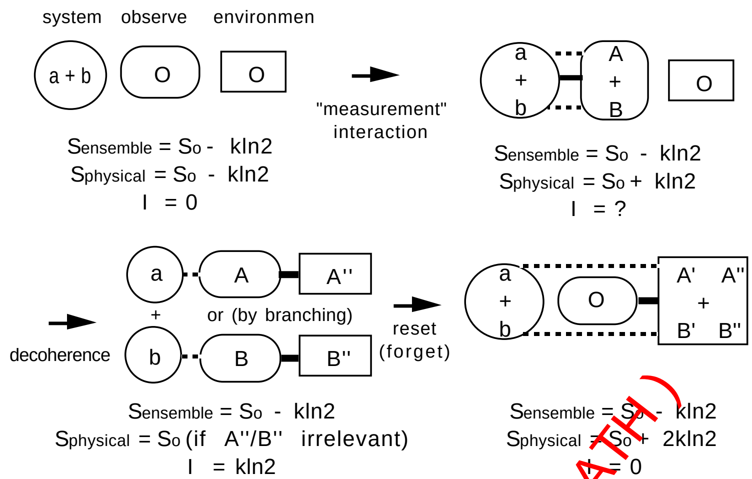

Fig. 2.3. Quantum measurement of a superposition by means of "branching" caused by decoherence (see text). The increase of physical entropy during the second step applies if the distinction between environmental degrees of freedom A''/B'', responsible for decoherence, is "irrelevant" (uncontrollable). After the last step, *all* entanglement has irreversibly become irrelevant in practice. Since the whole superposition is here assumed to "exist" forever (and may have future consequences *in principle*), the branching is meaningful only with respect to a *local* observer.

a macroscopic apparatus, this superposition remains nonetheless valid if Φ is defined to include the "rest of the universe", such as $|\Phi_n\rangle = |\phi_n\rangle|\chi_n\rangle$, with an environmental state $|\chi\rangle$. This powerful consequence of the Schrödinger equation holds regardless of all complications, such as decoherence and other, in practice irreversible, processes (which need not even be known). Therefore, it does seem that the measurement problem can only be resolved if the Schrödinger dynamics (2.1) is supplemented by a nonunitary collapse (2.2).

Specific proposals for such a process will be discussed in Chap. 8. Remarkably, however, there is no empirical evidence yet on where the Schrödinger equation may have to be modified for this purpose (see Joos 1986, Pearle and Squires 1994, or d'Espagnat 2001). On the contrary, the dynamical superposition principle has been confirmed with phantastic accuracy in spin systems (Weinberg 1989, Bollinger *et al.* 1989).

The Copenhagen interpretation of quantum theory insists that the measurement outcome has to be described in fundamental classical terms rather than as a quantum state. While according to Pauli (in a letter to Einstein: Born 1969), the appearance of an electron position is "a creation outside of the laws of Nature" (*eine ausserhalb der Naturgesetze stehende Schöpfung*), Ulfbeck and Bohr (2001) now claim (similar to Ludwig 1990 in his attempt to derive "the" Copenhagen interpretation from fundamental principles) that it is the *click in the counter* that appears "out of the blue", and "without an event that takes place in the source itself as a precurser to the click". Together with the occurrence of this, thus *not dynamically analyzable*, irreversible event in the counter, the wave function is then claimed to "lose its meaning" (precisely where it would otherwise describe decoherence!). The Copenhagen interpretation is often hailed as the greatest revolution in physics, since it rules out the general applicability of the concept of objective physical reality. I am instead inclined to regard it as a kind of "quantum voodoo": irrationalism in place of dynamics. The theory of decoherence describes events in the counter by means of a universal Schrödinger equation as a fast and for all practical purposes irreversible *dynamical* creation of entanglement with the environment (see also Shi 2000). In order to remain "politically correct", some authors have recently even *re-defined* complementarity in terms of entanglement (cf. Bertet *et al.* 2001), although the latter has never been a crucial element of the Copenhagen interpretation.

The "Heisenberg cut" between observer and observed has often been claimed to be quite arbitrary. This cut represents the borderline at which the probability interpretation for the occurrence of events is applied. However, shifting it too far into the microscopic realm would miss the readily observed quantum aspects of certain large systems (SQUIDs etc.), while placing it beyond the detector would require the latter's decoherence to be taken into account anyhow. As pointed out by John Bell (1981), the cut has to be placed "far enough" from the measured object in order to ensure that our limited capabilities of investigation (such as those of keeping the measured system isolated) prevent us from discovering any inconsistencies with the assumed classical properties or a collapse.

As noticed quite early in the historical debate, the cut may even be placed deep into the human observer, whose consciousness, which may be located in the cerebral cortex, represents the final link in the observational chain. This view can be found in early formulations by Heisenberg, it was favored by von Neumann, later discussed by London and Bauer (1939), and again supported by Wigner (1962), among others. It has even been interpreted as an *objective* influence of consciousness on physical reality (e.g. Wigner *l.c.*), although it may be consistent with the formalism only when used with respect to *one* final observer, that is, in a strictly subjective (though partly objectivizable) sense (Zeh 1971).

The "undivisible chain between observer and observed" is physically represented by a complex interacting medium, or a chain of intermediary systems $|\chi^{(i)}\rangle$, in quantum mechanical terms symbolically written as

$$
\begin{aligned}
&\left|\psi_n^{\text{system}}\right\rangle \left|\chi_0^{(1)}\right\rangle \left|\chi_0^{(2)}\right\rangle \cdots \left|\chi_0^{(K)}\right\rangle \left|\chi_0^{\text{obs}}\right\rangle \\
\to &\left|\psi_n^{\text{system}}\right\rangle \left|\chi_n^{(1)}\right\rangle \left|\chi_0^{(2)}\right\rangle \cdots \left|\chi_0^{(K)}\right\rangle \left|\chi_0^{\text{obs}}\right\rangle \\
&\vdots \\
\to &\left|\psi_n^{\text{system}}\right\rangle \left|\chi_n^{(1)}\right\rangle \left|\chi_n^{(2)}\right\rangle \cdots \left|\chi_n^{(K)}\right\rangle \left|\chi_n^{\text{obs}}\right\rangle \quad ,
\end{aligned} \tag{2.6}
$$

instead of the simplified form (2.4). This chain is thus assumed to act dynamically step by step (cf. Zeh 1973). While an initial superposition of the observed system now leads to a *superposition* of such product states (similar to (2.5)), we know empirically that a collapse must be "taken into account" by the conscious observer before (or at least when) the information arrives at him as the final link. If there are several chains connecting observer and observed (for example via other observers, known as "Wigner's friends"), the correctly applied Schrödinger equation warrants that each individual component (2.6) describes consistent ("objectivized") measurement results. From the subjective point of view of the final observer, all intermediary systems ("Wigner's friends" or "Schrödinger's cats", including their environments) could well remain in a superposition of drastically different situations until he observes (or communicates with) them!

Environment-induced decoherence means that an avalanche of other causal chains unavoidably branch off from the intermediary links of the chain as soon as they become macroscopic (see Chap. 3). This might even trigger a *real* collapse process (to be described by hypothetical dynamical terms), since the many-particle correlations arising from decoherence would render the total system prone to such as yet unobserved, but nevertheless conceivable, nonlinear many-particle forces (Pearle 1976, Diósi 1985, Ghirardi, Rimini, and Weber 1986, Tessieri, Vitali, and Grigolini 1995; see also Chap. 8). Decoherence by a *microscopic* environment has been experimentally confirmed to be *reversible* in what is now often called "quantum erasure" of a measurement (see Herzog et al. 1995). In analogy to the concept of particle creation, reversible decoherence may be regarded as "virtual decoherence". "Real" decoherence, which gives rise to the familiar classical appearance of the macroscopic world, is instead characterized by its unavoidability and irreversibility *in practice*.

In an important contribution, Tegmark (2000) was able to demonstrate that neuronal and other processes in the brain also become quasi-classical because of environmental decoherence. (Successful neuronal models are indeed classical.) This seems to imply that at least *objective* aspects of human thinking and behavior can be described by conceptually classical (though not necessarily deterministic) models of the brain. However, since no precise "localization of consciousness" within the brain has been found yet, the neural network (just as the retina, say) may still be part of the "external world" with respect to the unknown ultimate observer system (Zeh 1979). Because of Tegmarks arguments, this problem may not affect an *objective* theory of observation any longer.

Even "real" decoherence in the sense of above must be distinguished from a genuine collapse, which is defined as the *disappearance* of all but one components from reality (thus representing an irreversible *law*).[8] As pointed out above, a collapse could well occur much later in the observational chain than decoherence, and possibly remain less fine-grained. Nonetheless, it should then be detectable in other situations if its dynamical rules are defined. Environment-induced decoherence (the dynamically arising strong correlations with the rest of the world) leads to the important consequence that, in a world with no more than few-particle forces, robust *factor* states $|\chi_n^{\mathrm{obs}}\rangle$ are not affected by what goes on in the other branches that have formed according to the Schrödinger equation.

In order to represent a subjective observer, a physical system must be in a definite state with respect to properties of which he/she/it is aware. The salvation of a psycho-physical parallelism of this kind was von Neumann's main argument for the introduction of his "first intervention" (2.2): the collapse. As a consequence of the above-discussed dynamical independence of the different individual components of type (2.6) in their superposition, one may instead associate *all* arising factor wave functions $|\psi_n^{\mathrm{obs}}\rangle$ (different ones in each component) with *separate* subjective observers (that is, with different states of consciousness). This approach, which avoids a collapse as a new dynamical law, is essentially identical with Everett's "relative state interpretation" (so called, since the worlds *observed* by these observer states are described by their corresponding relative factor states). Although also called a "many worlds interpretation", it describes *one* quantum universe. Because of its (essential and non-trivial) reference to conscious observers, it may more appropriately be called a "multi-consciousness" or "many minds interpretation" (Zeh 1970, 1971, 1979, 1981, 2000, Albert and Loewer 1988, Lockwood 1989, Squires 1990, Stapp 1993, Donald 1995, Page 1995).[9]

[8] Proposed decoherence mechanisms involving event horizons (Hawking 1987, Ellis, Mohanty and Nanopoulos 1989) would either *require* a fundamental violation of unitarity, or merely represent a specific kind of environmental decoherence (entanglement beyond the horizon). The most immediate consequence of quantum entanglement is that quantum theory can be consistently applied only to the whole universe.

[9] As Bell (1981) pointed out, Bohm's theory would instead require consciousness to be psycho-physically coupled to certain *classical* variables (which this theory *postulates* to exist). These variables are probabilistically related to the wave function by means of a conserved statistical initial condition. Thus one may argue that the "many minds interpretation" merely eliminates Bohm's unobservable and therefore meaningless intermediary classical variables and their trajectories from this psycho-physical connection. This is possible because of the dynamical autonomy of a wave function that evolves in time according to a universal Schrödinger equation, and independently of Bohm's classical variables. These

Because of their dynamical independence, all these different observers (or rather, different "versions" of the same observer) cannot find out by means of experiments whether or not the other components have survived. This *consequence of the Schrödinger equation* thus leads to the *impression* (for separate observers) that all "other" components have "hurried out of existence" as soon as decoherence has become irreversible for all practical purposes. Then it remains a pure matter of taste whether Occam's razor is applied to the wave function (by adding appropriate but not directly detectable collapse-producing nonlinear terms to its dynamical law), or to the dynamical law (by instead adding myriads of unobservable Everett components to our conception of "reality"). Traditionally (and mostly successfully), consistency of the law has been ranked higher than simplicity of the facts.

Fortunately, the dynamics of decoherence can be discussed without giving an answer to this question. A collapse (real or apparent) has to be *taken into account* regardless of its interpretation in order to describe the dynamics of that wave function which represents our *observed quasi-classical world* (the time-dependent component which contains "our" observer states $|\chi_n^{\mathrm{obs}}\rangle$). Only *specific* dynamical collapse models could be confirmed or ruled out by experiments, while Everett's relative states, on the other hand, depend *in principle* on the definition of the observer system.[10] (No other "systems" have to be specified in principle. Their density matrices, which describe decoherence and quasi-classical concepts, are merely convenient.)

In contrast to Bohm's theory or stochastic collapse models, nothing has been said yet (or postulated) about *probabilities* of measurement outcomes. For this purpose, the Everett branches have to be given statistical weights in a way that appears *ad hoc* again. However, these probabilities are meaningful to an observer only as frequencies in *series* of equivalent measurements. These measurements must be performed in his branch (and would in general be performed and lead to different results in other branches). Graham (1970) was able to show that the norm of the superposition of *all those* Everett branches (arising from such series of measurements) which contain frequencies of results that substantially *differ* from Born's probabilities vanishes in the limit of infinite series. Although the definition of the norm, which is used in this argument, is exactly equivalent to Born's probabilties, it can be selected

variables thus cannot, by themselves, carry memories of their "surrealistic" history. Memory is solely in the quasi-classical wave packet that effectively guides them, while the other myriads of "empty" Everett world components (criticized for being "extravagant" by Bell) *exist* as well in Bohm's theory. Barbour (1994, 1999), in his theory of timelessness, effectively proposed a *static* version of Bohm's theory, which eliminates the latter's formal classical trajectories even though it preserves a concept of memories without a history ("time capsules" – see also Chapt. 6 of Zeh 2001).

[10] Another aspect of this observer-relatedness of the observed world is the concept of a *presence*, which is not part of *physical* time. It reflects the *empirical fact* that the subjective observer is local in space *and* time.

against other definitions of a norm by its unique property of being conserved under the Schrödinger dynamics.

To give an example: an isolated decaying quantum system may be described as a superposition of its metastable initial state and the outgoing channel wave function(s) for all its decay products. On a large but finite region of space and time the total wave function may then *approximately* decrease exponentially and coherently in accordance with the Schrödinger equation (with very small late-time deviations from exponential behavior caused by the dispersion of the outgoing waves). For a system that decays by emitting photons into a reflecting cavity, a *superposition of different decay times* has in fact been confirmed in the form of coherent "state vector revival" (Rempe, Walther and Klein 1987). An even more complex experiment exhibiting coherent state vector revival was performed by means of spin waves (Rhim, Pines and Waugh 1971). In general, however, the decay fragments would soon be "monitored" by surrounding matter. The resulting state of the environment must then depend (contain "information") on the decay time, and the superposition will decohere into dynamically independent components corresponding to *different* (approximately defined) decay times. From the point of view of a local observer, the system may be assumed to *have decayed at a certain time* (within the usually very narrow limits set by the decoherence time scale – see Sect. 3.3.2), even though he need not have observed the decay. This situation does not allow coherent state vector revival any more. Instead, it leads to an exponential distribution of decay times in the arising apparent ensemble, valid shortly after the decaying state has been produced (see Joos 1984).

However, as long as the information has not yet reached the observer,

$$\left(\sum_n c_n \left|\psi_n^{\rm system}\right\rangle \left|\chi_n^{(1)}\right\rangle \left|\chi_n^{(2)}\right\rangle \cdots \left|\chi_n^{(K)}\right\rangle\right) \left|\chi_0^{\rm obs}\right\rangle , \qquad (2.7)$$

he may as well assume (from his subjective point of view) that the nonlocal superposition still exists. According to the formalism, Schrödinger's cat (represented by $\left|\chi^{(2)}\right\rangle$, say) would then "become" dead *or* alive only when he becomes aware of it. On the other hand, the property described by the state $|\psi_n^{\rm system}\rangle$ (just as the cat's status of being dead or alive, $\left|\chi_n^{(2)}\right\rangle$) can also be assumed to have become "real" as soon as decoherence has become irreversible in practice. Therefore, decoherence must also corrupt any controllable entanglement that would give rise to a violation of Bell's inequalities (as it does – see Venugopalan, Kumar and Gosh 1995). If, instead of taking notice of the result, the observer would decide to perform another measurement of the "system" (by using a new observational chain), he could not observe any interference between different n's, since, as a local system, he cannot perform the required global measurements. All predictions which this observer can check are consistent with the assumption that the system was in *one* of the states $|\psi_n^{\rm system}\rangle$ (with probability $|c_n|^2$) before the second measurement (see also Sect. 2.4). This justifies the interpretation that the cat is *determined* to die or not yet to die as soon as irreversible decoherence has occurred somewhere in the chain (which will in general be the case *before* the poison is applied).

2.4 Density Matrix, Coarse Graining, and "Events"

The theory of decoherence uses some (more or less technical) auxiliary concepts. Their physical meaning will be recalled and discussed in this section, as it is essential for a correct interpretation of what is actually achieved with this theory.

In classical statistical mechanics, *incomplete knowledge* about the real physical state of a system is described by "ensembles" of states, that is, by probability distributions $\rho(p,q)$ on phase space (in Fig. 2.1 symbolically indicated by areas of uniform probability). Such ensembles are often called "thermodynamic" or "macroscopic states". Mean values of state functions $a(p,q)$ (that is, physical quantities that are determined by the microscopic state p,q), defined with respect to this ensemble, are then given by the expression $\int dp\, dq\, \rho(p,q) a(p,q)$. The ensemble $\rho(p,q)$ itself could be recovered from the mean values of a complete set of state functions (such as all δ-functions), while a (smaller) set, that may be realized in practice, determines only a "coarse-grained" probability distribution.

If all states which form such an ensemble are assumed to obey the same Hamiltonian equations, their probability distribution ρ evolves according to the Liouville equation,

$$\frac{\partial \rho}{\partial t} = \{H\, , \rho\} \quad , \tag{2.8}$$

with Hamiltonian H and Poisson bracket $\{,\}$. However, this assumption would be highly unrealistic for a many-particle system. Even if the *fundamental* dynamics is assumed to be given, the *effective* Hamiltonian for the considered system depends very sensitively on the state of the "environment", which cannot be assumed to be known better than that of the system itself. Borel (1914) showed long ago that even the gravitational effect resulting from shifting a small rock at the distance of Sirius by a few centimetres would completely change the microscopic state of a gas in a vessel here on earth within seconds after the retarded field has arrived (see also Chap. 3). In a similar connection, Ernst Mach spoke of the "profound interconnectedness of things". This surprising result is facilitated by the enormous amplification of the tiny differences in the molecular trajectories, caused by the slightly different forces, during subsequent collisions with other molecules (each time by a factor of the order of the ratio of the mean free path over the molecular radius). Similarly, microscopic differences in the state of the gas will immediately disturb its environment, thus leading in turn to slightly different effective Hamiltonians for the gas, with in general grossly different ("chaotic") effects on the microscopic states of the original ensemble. This will induce strong *statistical* correlations of the gas with its environment, whose neglect would lead to an increase of ensemble entropy.

This *effective local dynamical indeterminism* can be taken into account (when calculating *forward* in time) by means of stochastic forces (using a *Langevin equation*) for the individual state, or by means of a corresponding *master equation* for an ensemble of states, $\rho(p,q)$. The increase of the local ensemble entropy is thus attributed to an uncertain effective Hamiltonian. In this way, statistical correlations with the environment are regarded as dynamically irrelevant for the *future* evolution. An example is Boltzmann's collision equation (where the arising irrelevant correlations are *intrinsic* to the gas, however). The justification of this time-asymmetric procedure forms a basic problem of physics and cosmology (Zeh 2001).

When appying the conventional quantization rules to the Liouville equation (2.8) in a formal way, one obtains the *von Neumann equation* (or *quantum Liouville equation*),

$$i\hbar\frac{\partial\rho}{\partial t} = [H\,,\rho] \quad , \tag{2.9}$$

for the dynamics of "statistical operators" or "density operators" ρ. Similarly, expectation values $\langle A\rangle = \mathrm{tr}(\mathrm{A}\rho)/\mathrm{tr}(\rho)$ of observables A formally replace mean values $\bar{a} = \int dp\,dq\,a(p,q)\rho(p,q)$ of the state functions $a(p,q)$. Expectation values of a *restricted* set of observables would again represent a generalized coarse graining for the density operators. The von Neumann equation (2.9) is unrealistic for similar reasons as is the Liouville equation, although quantitative differences may arise from the different energy spectra – mainly at low energies. Discrete spectra have relevant consequences for macroscopic systems in practice only in exceptional cases, while they often prevent mathematically rigorous proofs of ergodic or chaos theorems which are valid in excellent approximation. However, whenever quantum correlations do form in analogy to classical correlations (as is the rule), they lead to far more profound consequences than their classical counterparts.

In order to explain these differences, the concept of a density matrix has to be derived from that of a (pure) quantum state instead of being postulated by means of quantization rules. According to Sect. 2.2, the probability for a state $|n\rangle$ to be "found" in a state $|\alpha\rangle$ in a corresponding measurement is given by $|\langle n \mid \alpha\rangle|^2$. Its *mean* probability in an ensemble of states $\{|\alpha\rangle\}$ with probabilities p_α representing incomplete information about the initial state α is, therefore, $p_n = \sum p_\alpha |\langle n \mid \alpha\rangle|^2 = \mathrm{tr}\{\rho \mathrm{P_n}\}$, where $\rho = \sum |\alpha\rangle p_\alpha \langle\alpha|$ and $P_n = |n\rangle\langle n|$. This result remains true for general observables $A = \sum a_n P_n$ in place of P_n. The ensemble of wave functions $|\alpha\rangle$, which thus defines a density matrix as representing a *state of information*, need not consist of mutually orthogonal states, although the density matrix can always be diagonalized in terms of its eigenbasis. Its representation by a *general* ensemble of states is therefore far from unique – in contrast to a classical probability distribution. Nonetheless, the density matrix can still be shown to obey a von Neumann equation *if* all states contained in the ensemble are assumed to evolve according to the same unique Hamiltonian.

However, the fundamental nonlocality of quantum states means that the state of a local system does *not exist* in general: it cannot be merely unknown. Accordingly, there *is no* effective local Hamiltonian that would allow (2.9) to apply in principle (see Kübler and Zeh 1973). In particular, a time-dependent Hamiltonian would in general require a (quasi-)classical environment. This specific quantum aspect is easily overlooked when the density matrix is introduced axiomatically by "quantizing" a classical probability distribution on phase space.

Quantum nonlocality means that the generic state of a composite system ("system" and environment, say),

$$|\Psi\rangle = \sum_{m,n} c_{mn} \left|\phi_m^{\text{system}}\right\rangle \left|\phi_n^{\text{environment}}\right\rangle \quad , \tag{2.10}$$

does not factorize. The expectation values of all *local* observables,

$$A = A^{\text{system}} \otimes \mathbb{1}^{\text{environment}} \quad , \tag{2.11}$$

have then to be calculated by "tracing out" the environment,

$$\langle\Psi| A |\Psi\rangle \equiv \text{tr}\left\{\text{A} |\Psi\rangle \langle\Psi|\right\} = \text{trace}_{\text{system}}\left\{\text{A}^{\text{system}} \rho^{\text{system}}\right\} \quad . \tag{2.12}$$

Here, the density matrix ρ^{system}, which in general has nonzero entropy even for a pure (completely defined) global state $|\Psi\rangle$, is given by

$$\begin{aligned} \rho^{\text{system}} &\equiv \sum_{m,m'} \left|\phi_m^{\text{system}}\right\rangle \rho_{mm'}^{\text{system}} \left\langle\phi_{m'}^{\text{system}}\right| \\ &:= \text{trace}_{\text{env}}\left\{|\Psi\rangle\langle\Psi|\right\} \equiv \sum_{m,m'} \left|\phi_m^{\text{system}}\right\rangle \sum_n c_{mn} c_{m'n}^* \left\langle\phi_{m'}^{\text{system}}\right| . \end{aligned} \tag{2.13}$$

It represents the specific *coarse-graining* with respect to all *subsystem* observables only. This "reduced density matrix" can be *formally* represented by various ensembles of local states (including its eigenrepresentation or diagonal form), although it does here characterize one pure but entangled *global* state. A density matrix thus based on entanglement has been called an "improper mixture" by d'Espagnat (1966). It can evidently not explain *ensembles* of definite measurement outcomes. If proper and improper mixtures were identified for operationalistic reasons (that are *based on* the measurement postulate), then decoherence would indeed completely "solve" the measurement problem.

Regardless of its origin and interpretation, the density matrix can be replaced by its partial Fourier transform, known as the *Wigner function* (see also Sect. 3.2.3):

$$
\begin{aligned}
W(p,q) &:= \frac{1}{\pi}\int e^{2ipx}\rho(q+x,q-x)\,dx \\
&\equiv \frac{1}{2\pi}\int\int \delta\left(q-\frac{z+z'}{2}\right)\mathrm{e}^{\mathrm{i}p(z-z')}\rho(z,z')\,dzdz' \\
&=: \mathrm{trace}\{\Sigma_{p,q}\rho\} \quad . \qquad (2.14)
\end{aligned}
$$

The second line is here written in analogy to the Bloch vector, $\pi_i = \mathrm{trace}\{\sigma_\mathbf{i}\rho\}$, since

$$
\Sigma_{p,q}(z,z') := \frac{1}{2\pi}e^{\mathrm{i}p(z-z')}\,\delta\left(q-\frac{z+z'}{2}\right) \qquad (2.15)
$$

is a generalization of the Pauli matrices (with index p,q instead of $i=1,2,3$ – see also Sect. 4.4 of Zeh 2001). Although the Wigner function is *formally* analogous to a phase space distribution, it does, according to its derivation, *not* represent an ensemble of classical states (phase space points). This is reflected by its potentially negative values, while even a Gaussian wave packet, which does lead to a non-negative Wigner function, is nonetheless *one* (pure) quantum state.

The degree of entanglement represented by an improper mixture (2.13) is conveniently measured by the latter's formal entropy, such as the linear entropy $S_{lin} = \mathrm{trace}(\rho - \rho^2)$. In a "bipartite system", the *mutual* entanglement of its two parts may often be controlled and *used* for specific applications (EPR-Bell type experiments, quantum cryptography, quantum teleportation, etc.). This is possible as far as the entanglement is not obscured by a mixed state of the total system. Therefore, other measures have been proposed to characterize the *operationally available* entanglement in mixtures (Peres 1996, Vedral *et al.* 1997). However, these measures do not represent the true and complete entanglement, since a mixed state, which reduces this measure, is either based on entanglement itself (on that of the whole bipartite system with its environment), or the consequence of averaging over an ensemble of unknown (but nonetheless entangled) states.

The eigenbasis of the reduced density matrix can be used, by othogonalizing the correlated "relative states" of the environment, to write the total state as a single sum,

$$
|\Psi\rangle = \sum_k \sqrt{p_k}\left|\hat{\phi}_k^{\mathrm{system}}\right\rangle\left|\hat{\phi}_k^{\mathrm{environment}}\right\rangle \quad . \qquad (2.16)
$$

While Erhard Schmidt (1907) first introduced this representation as a mathematical theorem, Schrödinger (1935) used it for discussing entanglement as representing what he called "probability relations between separated systems". It was later shown to be useful for describing quantum nonlocality and decoherence by means of a universal wave function (Zeh 1971, 1973).

The two orthogonal systems $\hat{\phi}_k$ of the Schmidt form (2.16) are *determined* (up to degeneracy of the p_k's) by the total state $|\Psi\rangle$ itself. A time dependence of $|\Psi\rangle$ must therefore affect both the Schmidt states *and* their (formal) probabilities p_k (see Kübler and Zeh 1973, Pearle 1979, Albrecht 1993), hence also the subsystem entropy, such as the linear entropy $\sum p_k(1-p_k)$. The induced subsystem dynamics is thus not "autonomous". Similar to the motion of a shadow that merely reflects the regular motion of a physical object, the reduced information content of the subsystem density matrix by itself is insufficient to determine its own change. Likewise, Boltzmann had to introduce his *Stoßzahlansatz*, based on statistical assumptions, when neglecting *statistical* correlations between particles (instead of quantum entanglement). The exact dynamics of any local "system" would in general require the whole Universe to be taken into account.

Effective "open systems quantum dynamics" has indeed been *postulated* in analogy to the Boltzmann equation by means of semigroups or master equations for calculating forward in time. An equivalent formalism was introduced by Feynman and Vernon (1963) in terms of path integrals. As explained above, this description can neither be exact, nor would it justify the replacement of improper mixtures by proper ones *unless explicitly postulated as a fundamental correction to the Schrödinger equation.* The formal theory of master equations will be discussed in Chap. 7 (see also Zeh 2001). Its foundation for local systems in terms of a global unitary Schrödinger equation requires very specific (statistically improbable) cosmic initial conditions.

According to a universal Schrödinger equation, quantum correlations with the environment are permanently created with great efficiency for all macroscopic systems, thus leading to decoherence, defined as the irreversible dislocalization of phase relations (see Chap. 3 for many examples). The apparent (or "improper") ensembles, obtained for subsystems in this way, often led to claims that decoherence be able (or meant) to solve the measurement problem.[11] The apparent nature of these ensembles has then in turn been used to declare the program of decoherence a failure. As explained in Sect. 2.3, both claims miss the point. However, decoherence represents a *crucial dynamical step* in the measurement process. The rest may remain a pure epistemological problem (requiring only a reformulation of the psycho-physical parallelism in quantum mechanical terms). If the Schrödinger equation is exact, the observed quantum indeterminism can only reflect that of the observer's identity – not one to be found in objective dynamics.

The process of decoherence leads to a novel, dynamically consistent concept of a generalized course graining. If phase relations between certain sub-

[11] In the Schmidt basis, interference terms are *exactly* absent by definition. Hepp (1972) used the formal limit $N \to \infty$ to obtain this result in a *given* basis (while this may require infinite time). However, the global state always remains one pure superposition. The Schmidt representation has therefore been used instead to specify the Everett branching, that is, to define the ultimate "pointer basis" $|\chi_n^{\text{observer}}\rangle$ for each observer (cf. Zeh 1973, 1979, Albrecht 1992, 1993, Barvinsky and Kamenshchik 1995). It is also used in the "modal interpretation" of quantum mechanics (cf. Dieks 1995).

spaces of a system permanently disappear by decoherence, their reduced density matrices may be approximated in the form

$$\rho = \sum_{m,n} P_m \rho P_n \approx \sum_n P_n \rho P_n \quad , \tag{2.17}$$

where P_n projects on to the n-th decohered subspace, while $\sum P_n = \mathbb{1}$. The dynamics of the formal probabilities $p_n(t) := \mathrm{tr}\{P_n \rho(t)\}$ may then be written as a master equation, similar to the Pauli equation,

$$\dot{p}_n = \sum_m A_{nm}(p_m - p_n) \quad , \tag{2.18}$$

as was shown by Joos (1984). Since (2.18) describes stochastic subsystem dynamics (in the direction of time that is characterized by the process of decoherence), it defines probabilities for coarse-grained "histories" $n(t)$, corresponding to time-ordered sequences of projections $P_{n_1}(t_1) \ldots P_{n_k}(t_k)$. Probabilities for such histories in discrete time steps can be written as

$$p(n_1, \ldots, n_k) = \mathrm{tr}\{P_{n_k}(t_k) \ldots P_{n_1}(t_1)\rho(t_0)\} \tag{2.19}$$

(using of the property $P^2 = P$ of projection operators). States n_k dynamically arising according to a master equation may contain "consistent memories" (or "time capsules" in Barbour's words), while the corresponding apparent histories appear "quasi-classical" (robust under decoherence). Such histories would individually obey a *quantum Langevin* equation (an indeterministic generalization of the Schrödinger equation). Models (often assumed to hold *exactly* instead of the Schrödinger equation) have been proposed by Diósi (1986), Belavkin (1988), Gisin and Percival (1992), and others – see also Diósi and Kiefer (2001).

In the theory of "consistent histories" (Griffiths 1984, Omnès 1992, 1995), *formal* projections P_n are called "events" regardless of any dynamics. These events are thus *not* dynamically described within the theory – in accord with the Copenhagen interpretation, where events are assumed to occur "out of the blue" or "outside the laws of nature". However, only those histories $n_1, \ldots n_k$ are then *admitted by postulate* (that is, assumed to "occur") which possess "consistent" probabilities – in the dynamical sense of being compatible with a stochastic *evolution*. This condition *requires* (a weak form of) decoherence, which is *not* generally based on entanglement (cf. Omnès 1999). This dynamical dilemma is then resolved by Griffiths and Omnès by introducing a "new logic". It culminates in Omnès' (1995) surprising remark that "the formalism of logic is not time-reversal invariant, as can be seen in the time ordering of the (projection) operators". However, a property a at time t_1 that is said to "imply" a property b at time $t_2 > t_1$ would describe a *causal* (that is, dynamical) rather than a *logical* relationship. This conceptual confusion of cause and reason seems to have a long tradition in philosophy, while even in mathematics the truth of logical theorems is often inappropriately *defined* by means of logical *operations* that have to be performed in time (thus mimicking a causal relation).

In the theory of decoherence, *apparent* events in the detector are described dynamically by the universal Schrödinger equation, using certain initial conditions for the environment, as a very fast but smooth formation of entanglement. Similarly, "particles" *appear* in the form of narrow wave packets in space as a consequence of decoherence in the detector. This identification of observable events with a decoherence process holds regardless of any conceivable subsequent genuine collapse. Therefore, *decoherence is not only responsible for the classical aspects of quantum theory, but also for its "quantum" aspects* (see Sect. 3.3.2.3). All fundamental physical concepts are continuous and based on "smooth" Schrödinger dynamics.

This description of quantum events as a unitary process also avoids any "superluminal effects" that have been shown (with mathematical rigour – see Hegerfeldt 1994) to arise (not very surprisingly) from explicitly or tacitly assumed instantaneous quantum jumps between exact energy or particle number eigenstates.[12] The latter require infinite exponential tails that can never *form* completely in a relativistic world. Supporters of explicit collapse mechanisms are quite aware of this problem, and try to avoid it (cf. Diósi and Lukácz 1994 and Chap. 8). In the nonlocal quantum formalism, *dynamical locality* is achieved by using Hamiltonian operators that are spatial integrals over a Hamiltonian density. This form prevents superluminal signalling and the like.

2.5 Conclusion

Let me recall the interpretation of quantum theory that has now emerged in accordance with the concept of decoherence:
(1) General quantum superpositions (such as a wave function) represent individual physical states (Sect. 2.1.1).
(2) According to a universal Schrödinger equation, most superpositions are almost immediately, and in practice irreversibly, *dislocalized* by interaction

[12] Such superluminal "phenomena" are reminiscent of the story of *Der Hase und der Igel* (the race between *The Hedgehog and the Rabbit*), narrated by the Grimm brothers. Here, the hedgehog, as a competitor in the race, does not run at all, while his wife is waiting at the end of the furrow, shouting in low German "Ick bin all hier!" ("I'm already here!"). Similar arguments hold for "quantum teleportation" (cf. also Vaidman 1998). Experiments clearly support the view that reality is described by a smoothly evolving wave function, nonlocal but dynamically compatible with relativity, rather than in terms of probabilistic "events" (cf. Fearn, Cock, and Milonni 1995). It is the local observer whose identity "splits" indeterministically according to the Schrödinger equation. Superluminal teleportation *would* be required to describe the corresponding experiments if (local) physical properties entered existence "out of the blue" in fundamental quantum events.

with their environment. Although the resulting nonlocal superpositions still *exist*, we do not know, in general, what they *mean* (or how they could be observed). However, if dynamics is local (described by a Hamiltonian density in space, $H = \int h(\mathbf{r})d^3\mathbf{r}$), approximately factorizing components may be dynamically autonomous after this decoherence has occurred, and nonlocal superpositions cannot return into local ones if statistical arguments apply to the future (Zeh 2001).
(3) Any observer (assumed to be local for empirical and dynamical reasons) who attempts to observe a subsystem of the nonlocal superposition must become part of this entanglement. Those of his component states which are then related only by nonlocal phase relations describe different observations. So we may axiomatically identify these individual *component* states of the observer with states of consciousness (novel psycho-physical parallelism).
(4) Because of this dynamical autonomy of decohered world components ("branches"), there is no reason to deny the existence of "the other" components which result from the Schrödinger equation ("many minds interpretation" – Sect. 2.3).
(5) Probabilities are meaningful only as frequencies in series of repeated measurements. In order to derive the observed Born probabilities in terms of frequencies, we have to *postulate* merely that we are living in an "Everett branch" with not extremely small norm.

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