

In these lectures, the author's point of view on the problem of Hawking Evaporation of Black Holes is explained in some detail. A possible resolution of the information loss paradox is proposed, which is fully in accord with the rules of quantum mechanics. Black hole formation and evaporation leaves over a remnant which looks pointlike to an external observer with low resolving power, but actually contains a new infinite asymptotic region of space. Information can be lost to this new region without violating the rules of quantum mechanics. However, the thermodynamic nature of black holes can only be understood by studying the results of measurements that probe extremely small (sub- Planck scale) distances and times near the horizon. Susskind's description of these measurements in terms of string theory may provide an understanding of the Bekenstein-Hawking (BH) entropy in terms of the states of *stranded strings* that cross the horizon. The extreme nonlocality of string theory when viewed at short time scales allows one to evade all causality arguments which pretend to prove that the information encoded in the BH entropy can only

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be accessed by the external observer in times much longer than the black hole evaporation time. The present author believes however that the information lost in black hole evaporation is generically larger than the BH entropy, and that the remaining information is causally separated from the external world in the expanding horn of a black hole remnant or *cornucopion*. The possible observational signatures of such cornucopions are briefly discussed.

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1. Introduction - The Facts In the Case

The subject that I am going to talk about in these lectures, the Hawking Evaporation of Black Holes, has been with us for on the order of nineteen years now. Although the last few years have seen an upsurge of interest and activity in the subject, it remains a frustrating field which stubbornly refuses to yield a satisfying resolution of its paradoxes. The number of very good physicists who have expressed fairly definitive opinions about the resolution of the Hawking puzzle is smaller than the number of definitive opinions they have expressed. The frustration is compounded by the fact that there is no hope for experimental resolution of the confusion. I would guess that there is still a sizable body of physicists and astronomers who remain unconvinced of the observational evidence for the existence of black holes. Given that they exist, the probability that we are ever going to examine a black hole close up seems very small. Even if we could examine one close up the probability they we would happen to observe it at a time when it was emitting substantial amounts of Hawking radiation is nil. And most frustrating of all, even those physicists who insist that black hole radiation is not “really” incoherent, agree that it looks like thermal radiation for all practical purposes.

Much of the recent activity and excitement has centered on two dimensional models in which it was hoped that one could examine the black hole puzzle in a mathematically controlled context. Although this hope has so far proved illusory, I believe that some progress has been made. One is however cautioned that at least some of the contributors to the field feel that these models do not capture enough of the “real” problem to resolve the paradoxes. This opinion is based on the belief that the apparent paradoxes of Hawking radiation are pointing us toward fundamental clues about the nature of quantum gravity. Thus one may argue that only the full theory of the real world will give a satisfactory account of the resolution of these paradoxes; models will not suffice. I do not agree with this point of view, although as I will indicate below, I am not completely in disagreement either.

The title of this section implies a certain degree of objectivity. In a field such as this, true objectivity is impossible. I will therefore be presenting “the facts” in a way which emphasizes that part of the data that supports my present opinions. It is best then to get

those opinions out on the table, so that you can judge for yourself what they are worth and how much they are distorting my presentation of the facts. In brief: I believe that the Hawking evaporation of black holes terminates in stable remnants. An angular slice of the geometry of those remnants is shown in Fig. 1.

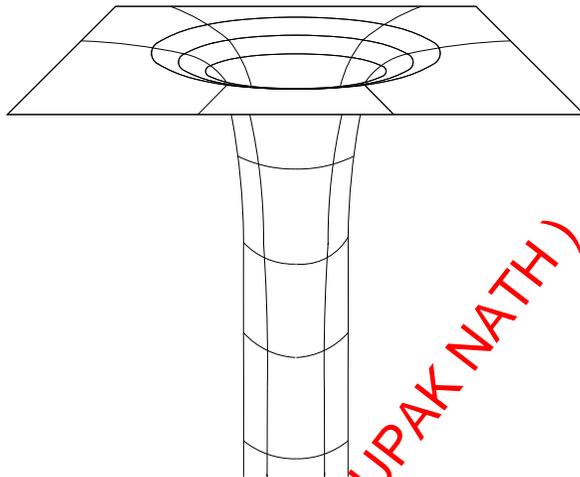


Fig. 1: An Angular Slice of the Static Geometry of a Black Hole Remnant

It is a small “hole in space” attached to a semi-infinite horn which has a spherical cross section of small radius. These static geometries have a unique quantum ground state, but they are the remains of evolving geometries possessing a horizon which moved off to infinite spacelike distance. The Penrose diagram of the full spacetime of one of these remnants is shown in Fig. 2.

There can be many different quantum states propagating behind the horizon of these remnants, but the external geometry and ADM mass of all of these states is the same. This is the repository for all of the information “lost” in Hawking evaporation. The full spacetime has (in the approximation in which quantum fluctuations of the gravitational field can be treated perturbatively) a unitary quantum mechanical evolution, but the past and future contain different numbers of causally separated spatial asymptotic regions. The S matrix for transitions from the initial asymptotic region back to itself is not unitary.

This by itself may seem like a resolution of the information loss puzzle, and for a while

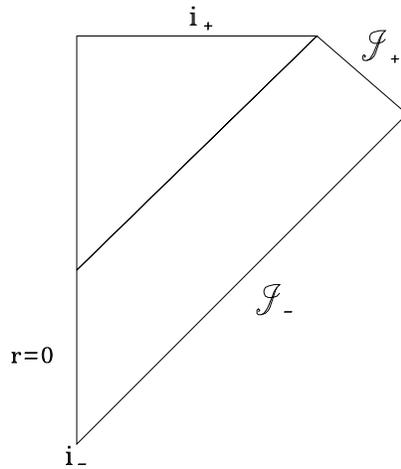


Fig. 2: The Penrose Diagram of a Black Hole Remnant

I believed that it was. However, neither it nor any other discussion of information that is truly lost to the external observer, can account for the Hawking Bekenstein entropy. This is a true thermodynamic entropy which describes interactions between the black hole and the external observer. As such, it must be associated with degrees of freedom which are causally connected to, and can interact with, the external observer. By assigning the entropy to correlations with degrees of freedom which are out of causal contact with the external observer, or in a topologically disconnected universe, one gives up the possibility of explaining its thermodynamic nature. I will argue in more detail below that the location of the Bekenstein-Hawking degrees of freedom must be in an extremely tiny region in the vicinity of the horizon of the black hole. In discussing them one is inevitably led into extremely short distance physics. I believe that Susskind's string model of these degrees of freedom, while still in a primitive stage of development, may lead to an ultimate explanation of the Bekenstein-Hawking entropy. If this is the case, then the information represented by the BH entropy is not lost to the external observer.

My picture of Hawking evaporation then, includes in some manner all of the current theoretical prejudices about the subject. I believe in remnants of black hole evaporation, but all information stored in them is causally inaccessible to the external observer. The S-matrix for a single asymptotic spatial region is not unitary once a black hole has formed.

However, the information whose absence is quantified by the Bekenstein-Hawking entropy is not irretrievably lost. It is located on the horizon of the black hole, probably in the form of *stranded strings* (see below for a definition), and will be radiated into the original asymptotic region as the black hole evaporates.

1.1. Some Classical Facts

For the author at least, the only way to get intuition about what is going on in general relativity is to work in synchronous gauge. This a name for any one of a collection of coordinate systems in which “time is time”, and general relativity is a theory of the dynamics of spatial geometry. A synchronous gauge is chosen by picking a spacelike hypersurface and defining time to be the geodesic distance orthogonal to this hypersurface. For a typical hypersurface, synchronous gauge may not be well defined on the entire spacetime manifold, but for the Schwarzschild black hole there are two useful synchronous coordinate systems that tell us what is going on. The simplest of these is related to internal Killing coordinates by a simple reparametrization of the time variable. The metric takes the form

$$ds^2 = -dt^2 + \left(\frac{2GM}{\tau} - 1\right)dr^2 + \tau^2 d\Omega^2 \quad (1.1)$$

$$\frac{t}{2GM} = \frac{\pi}{2} - \arccos\left(-\sqrt{\frac{\tau}{2GM}}\right) - \sqrt{\frac{\tau}{2GM}\left(1 - \frac{\tau}{2GM}\right)} \quad (1.2)$$

$$\frac{\tau}{2GM} \sim \left(\frac{t}{2GM}\right)^{\frac{2}{3}} \quad : t \rightarrow 0 \quad (1.3)$$

$$0 \geq \frac{t}{2GM} \geq -\frac{\pi}{2}; \quad 0 \leq \tau \leq 2GM \quad (1.4)$$

This coordinate system covers the inside of the Schwarzschild horizon. Its equal time surfaces are the hyperboloids with positive UV in Kruskal coordinates. It is convenient for pictorial purposes to modify the Schwarzschild metric by making the usual “dust” model of a collapsing star; sewing the geometry (1.1) onto a collapsing matter dominated Roberston Walker universe. The time evolution of the geometry of the spatial sections is shown in Fig. 3.

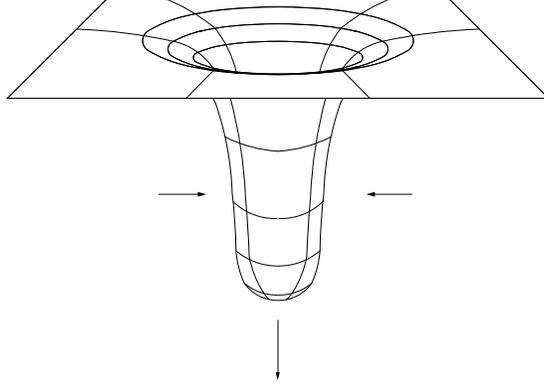


Fig. 3: The Spatial Geometry Inside a Schwarzschild Black Hole Evolves Into an Infinite, and Infinitely Thin, Horn

Note that the geometry has the “hornlike” shape of Fig. 1, but the horn is dynamic. It stretches along its length and shrinks transversely, reaching infinite length and zero width in finite proper time. This is the celebrated Schwarzschild singularity. It has two aspects: the width goes to zero, and the infinite length is achieved in finite time. Both of these correspond to curvature singularities.

From these pictures, it is easy to understand why the black hole has a horizon. In Einstein’s general relativity, space is allowed to stretch faster than light can cross it. A light beam sent out from some object may never be able to get back to its point of origin because in the time that it has travelled, the space has expanded (and continues expanding). Now let us introduce Novikov coordinates, a synchronous coordinate system that takes as its initial hypersurface the $t = 0$ surface of external Schwarzschild coordinates. This coordinate system in fact covers the whole of the black hole (including, though we will not use this fact, its Kruskal extension). Unfortunately the metric has the following ugly form in these coordinates

$$ds^2 = -dt^2 + \frac{(\rho^2 + 1)}{rho^2} \left(\frac{dR}{d\rho}\right)^2 d\rho^2 + R^2 d\Omega^2 \quad (1.5)$$

where $R(\rho, t)$ is a function defined by

$$\frac{t}{2GM} = \pm(\rho^2 + 1) \left[\frac{R}{2GM} - \frac{\left(\frac{R}{2GM}\right)^2}{\rho^2 + 1} \right]^{\frac{1}{2}} + (\rho^2 + 1)^{\frac{3}{2}} \arccos \left[\frac{R}{2GM(\rho^2 + 1)} \right]^{\frac{1}{2}} \quad (1.6)$$

Near $R = 0$, which is the line $\frac{t}{2GM} = \frac{\pi}{2}(\rho^2 + 1)^{\frac{3}{2}}$, $\frac{dR}{d\rho} \sim R^{-\frac{1}{2}}$. In Novikov coordinates the geometry of a collapsing star outside the horizon is relatively slowly varying and becomes asymptotically static, while the inner geometry resembles that found in (1.1). It is clear then that if we look at any point a finite distance inside the horizon on some Novikov time slice, the space between it and any point outside the horizon will expand so rapidly that light will not be able to travel between them. Indeed, in the Schwarzschild geometry, the radial expansion rate of the interior geometry in Novikov coordinates becomes infinite in finite proper time. Note however, that even if the singularity were absent, and the transverse size of the horn remained finite, we could still have superluminal expansion of the interior geometry and would be forced to conclude that the system had a horizon.

The synchronous view of the black hole interior suggests a possible scenario for its nonsingular evolution. Namely, all that is necessary is to find a mechanism for stabilizing the horn against transverse collapse, and of slowing down the expansion in the radial direction so that the rate never becomes infinite. We will see below that this is precisely what happens for extremal charged black holes. One thing is clear about any such hypothetical mechanism for stabilization of the black hole. If we believe (as we should) that the Schwarzschild solution for a large mass black hole is valid down to times at which the transverse dimension has shrunk to microscopic size then the radial extent of the geometry will be very large. In fact, as we shall see, the example of charged extremal black holes suggests that perhaps the geometry keeps on growing in the radial direction, even when the transverse collapse is averted. We are thus led to a picture of black hole final states as large one dimensional protrusions on the geometry of space which connect onto the space of the asymptotic observer through essentially pointlike openings. From many points of view, the external observer will regard these as point particles, but we will argue that this is a mistake when it comes to quantum mechanics.

We now turn to another well known classical feature of black holes, the discrepancy between the description of black hole physics given by infalling observers, and those, like the observer at infinity, who are supported in the gravitational field and remain outside the horizon. When the black hole mass is large, the infalling observer experiences nothing in particular as she falls through the horizon. To the asymptotic observer on the other

hand, the horizon is a very peculiar place. Nothing seems to fall through it. This can be ascribed to the behavior of the g_{00} component of the metric in Schwarzschild coordinates . Near the horizon the metric has the form

$$-\frac{x}{2GM}dt^2 + \frac{x}{2GM}^{-1}dx^2 + (2GM)^2d\Omega^2 \quad (1.7)$$

where $x = r - 2GM$. The proper time per unit Schwarzschild coordinate time goes to zero as the horizon is approached. This is a Lorentz contraction. A supported observer near the horizon is accelerating like mad to prevent himself from falling through. The instantaneous boost relating his frame to that of the infalling observer is extremely large, of order $e^{\frac{t}{4M}}$, where t is the Schwarzschild time coordinate. For a large mass black hole, even moderate energies will be boosted way above the Planck energy by such a transformation. Thus the Schwarzschild observer sees the infalling observer as highly Lorentz contracted. From his point of view the geodesic observer's clocks run very slowly and the structure in her machinery has very little extent in the radial direction .

Now suppose that the asymptotic observer has measuring apparatus of limited accuracy, which cannot measure arbitrarily small intervals of space or time. Very quickly, the infalling observer reaches a point at which the full extent of her apparatus in the radial direction is apparently squeezed into less than a minimally measurable distance from the horizon. All normal processes in the infalling observers frame are slowed down so much that the external observer cannot discern anything changing with time over the minimal interval between ticks of his clock. Thus, a classical observer with limited powers of observation, quickly loses all information about what is going on in the infalling observer's frame. The traditional "covariant" view of relativity is that this is a consequence of a bad choice of coordinates, but in the past few years a new paradigm has developed which takes the Schwarzschild observer's point of view as the basis of the treatment of the astrophysics of black holes, and their interactions with the external environment. The Membrane Paradigm[1], as it is called, claims that all interactions of the black hole with the external world can be treated correctly by a model in which the black hole behaves *as if* there were a physical membrane with charge, current and energy densities on it hanging on a timelike surface just above the horizon of the hole. The classical aspects of this picture can be

derived from general relativity, but it has also been applied to the treatment of Hawking radiation. We shall see that such a picture for Hawking radiation seems to contradict the axioms of quantum field theory, but that it may be derivable from the dynamics of strings.

There are two important features of this classical picture that we will want to remember later when we discuss Susskind's conjectures about the nature of the Bekenstein-Hawking Entropy. The first is that the Schwarzschild observer's attempts to understand what is happening to his infalling colleague as she approaches the horizon require him to contemplate measurements of arbitrarily small length and time intervals. Thus, a proper theory of these measurements requires us to understand physics at the shortest distances. Secondly, although the Schwarzschild observer's picture is an "incorrect" picture of what is "really happening" to the infalling observer, it is a perfectly sensible account of everything that the Schwarzschild observer can actually measure. In the end, I believe that this will be the sort of situation that we will recover for the quantum theory of black holes, at least close to the semiclassical limit of large mass. Our picture of "what is really going on" in Hawking evaporation will be the formation of a remnant and the disappearance of particles behind a receding horizon. However, the asymptotic observer will be able to account for much of what he observes in terms of a gas of "stranded strings" glued to the a membrane on the *stretched horizon*[2]. I will suggest below that there is probably some real information loss to the asymptotic observer, but that the thermodynamic entropy of Bekenstein and Hawking represents information that can in principle (though certainly not in practice) be retrieved by him.

1.2. Quantum Facts

Here I will briefly review the salient facts about the theory of Hawking radiation. I assume that the listener/reader is already familiar with this material and I will only emphasize some important facts that are not usually presented. Hawking's calculation is carried out in the framework of an approximation to quantum gravity called *quantum field theory in curved spacetime*. One imagines the formation of a black hole by a classical matter distribution falling in from infinity in an initially flat spacetime. The first quantum correction to this classical process consists of quantizing the linear fluctuations of quantum

fields around this classical solution. Hawking computed the S-matrix for this linear field theory. What does this involve? The classical geometry has a perfectly well defined past asymptotic region. The future however consists of two causally disconnected asymptotic regions (the original one, and a region “down the horn” in the synchronous gauge picture), one of which becomes singular a finite time in the future. Hawking’s idea was to treat the singular region “as if” it had a well defined set of asymptotic states. We can then consider *inclusive cross sections* in which the external observer measures only what is causally accessible to him, summing over the unknown final states on the other side of the horizon. As usual this will lead to a density matrix description of the final state that he measures.

The expansion parameter for this semiclassical approximation is the Planck mass divided by the mass of the black hole. In the limit of large mass the spacetime curvature is small (order $\frac{1}{(GM)^2}$) everywhere except for the singularity¹. Furthermore, the singularity is a large timelike geodesic distance (of order GM) from any point on the horizon. In assessing the size of corrections to the semiclassical approximation one must recognize that any quantum field theory (and most particularly quantum gravity) has ultraviolet divergent quantum fluctuations. One must have in mind the value of the physical cutoff if one is to estimate these. There are two important possible choices for the cutoff scale. One is the Planck mass itself. The other possibility is to assume the existence of a small dimensionless parameter which controls the magnitude of the short distance fluctuations of gravity. This is the prescription of weakly coupled string theory. In effect the second option is the claim that the true cutoff scale L_S , is larger than the Planck length, the small dimensionless loop expansion parameter being the square of the ratio $\frac{L_P}{L_S}$.

According to Hawking’s calculation[3] the expression for the outgoing density matrix is

$$\rho = e^{-\frac{16\pi^2 GMH}{h}} \tag{1.8}$$

where h is Planck’s constant. The black hole is thus seen to behave like a black body with temperature $T_H = \frac{h}{16\pi^2 GM}$. Since its energy is M , the first and second laws of

¹ In particular, it is small near the horizon.

thermodynamics give it an entropy $S_{BH} = \frac{8\pi^2 GM^2}{h}$ with the usual ambiguity of an additive constant. Note that this entropy is proportional to the area of the horizon $S_{BH} = \frac{1}{4}AM_P^2$. According to the Stefan Boltzmann Law, and taking into account that the area of the horizon is $\sim M^2$ the black hole should lose energy at a rate $\frac{dM}{dt} = -\frac{1}{M^2}$, giving it a lifetime of order M^3 . All of this is for a neutral nonrotating black hole. In general, the black hole temperature depends on its mass, angular momentum, and charge. In particular, for near extremal charged black holes the lifetime is linear in the deviation from extremality.

The paradox of all this is that the black hole seems to decay into incoherent radiation. Below we will review the argument that suggests that in standard quantum field theory the decay of the hole proceeds incoherently until a time when its energy content is very small compared to the amount of information that it has yet to liberate. This appears to lead to a choice between three alternative scenarios for the climax of the radiation process, all of which appear to lead to paradoxes. We will enumerate and discuss them below.

The other key question raised by this discussion is the origin of the Bekenstein-Hawking entropy $S_{BH} = \frac{1}{4}AM_P^2$. It does not seem to come from a counting of states. In particular, if we try to compute the information theoretic entropy of entanglement in the density matrix computed by Hawking, we find an infinite answer, coming from two different sources. One of these is silly. We have computed the density matrix as if the black hole metric remained static for all time, even though it is emitting thermal radiation. This gives us a trivial volume infinity. The other infinity is more interesting, and was first discussed by 't Hooft[4]. Its origin is in ultraviolet divergences. Conventional quantum field theory has an infinite number of states in any arbitrarily small volume of space time. Furthermore, the correlation functions of degrees of freedom at nearby points diverge as the distance between the points goes to zero. Thus if we choose a spacelike surface and make an imaginary line cutting it into two, the entropy of entanglement of the degrees of freedom on one side with those on the other, is divergent[5]. The coefficient of this divergence depends on how we choose to cut off the theory. 't Hooft and Susskind base their theories of black hole evaporation on the contention that this infinite correction to the Hawking calculation of the entropy of the density matrix is a signal of something wrong in the conventional field theoretic treatment of the problem. The goal of a proper theory,

according to them, is to understand the physical origin of the BH entropy in a context which allows one to compute the finite quantum corrections to the BH formula.

2. The Threefold Way

2.1. Subtle Correlations and Causality

At first sight, the most conservative approach to the problem of information loss is that which goes under the name of “subtle correlations”. According to this dogma, the S-matrix for black hole formation and evaporation is unitary in the Hilbert space of the original asymptotic observer. The apparent loss of coherence exhibited in Hawking’s calculation is ascribed to the inadequacy of his semiclassical approximation. The standard analogy is to the heating of a lump of coal: Suppose that we encode the information in the Encyclopedia Britannica in Morse code and send it out as flashes of laser light that are directed at a large lump of coal. The laser flashes are absorbed by the coal, which heats up. All of the information in the Encyclopedia is now contained in the coal. Of course the heated coal emits infrared radiation and eventually cools down. After it has cooled, the information is stored in the heat that it radiated, but for all practical purposes this radiation is thermal and the information has been lost.

In this situation we understand what is going on. The radiation from a cooling lump of coal is not really thermal, it is in a pure, albeit very complicated, quantum state. The useful information originally stored in the pulsed laser beam is now encoded in correlations between photons which were emitted from the coal at very different times, and are therefore very far from each other in space. This nonlocally stored information is of no practical use, and for local measurements, the pure state is equivalent to a mixed state. Is this all that is going on in the Hawking calculation?

There is a very strong argument that this is not the case, at least not within the conventional formalism of quantum field theory. The semiclassical picture of Hawking evaporation is valid for most of the evaporation of a large black hole². In particular, if we

² This contention has been vigorously challenged by ’t Hooft[6] and others[7]. The most compelling argument that it is correct comes from the large N analysis of two dimensional evaporating

have an enormous black hole which has evaporated away 99% of its mass, leaving behind a hole which is still large, the Hawking calculation will be accurate to the past of the asymptotically null spacelike slice labelled 99 in the Penrose diagram of Fig. 4.

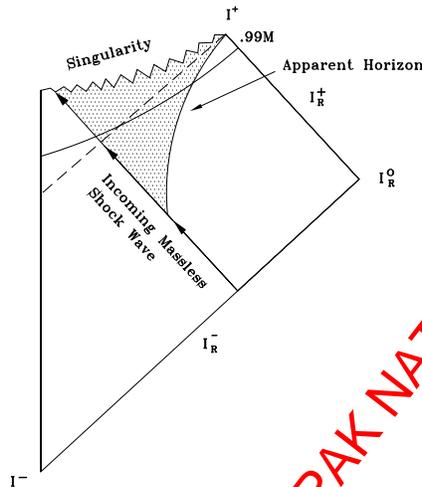


Fig. 4: Penrose Diagram Illustrating the Argument That Information Cannot Get Out of a Black Hole Until Times Longer than The Hawking Lifetime

We can examine the state of that portion of the world which is behind the horizon. In the semiclassical approximation, this will be calculable, and correlated with the state of the outside world. Observers falling through the horizon, but still not at the singularity, do not notice any particularly violent events. The interior state will be impure, with large entropy, determined by the correlations (in the initial state) between objects that fall into the black hole and those that don't. If we assume the usual rules of quantum mechanics and locality, this will also be the entropy of the state seen by an external observer. There do not seem to be any principles which would prevent this entropy of entanglement from being of order the Bekenstein-Hawking entropy of the initial hole, $\sim \frac{M^2}{M_P^2}$. Thus, within the domain of validity of the semiclassical approximation, we can establish the existence

black holes. The large N expansion appears to provide a systematic renormalizable expansion of all correlation functions in the system which is controlled by a small parameter in the entire region of the semiclassical spacetime to the past of the spacelike slice 99 described below. The expansion appears to break down only in a region near the singularity.

of a time slice on which the energy of the external system is small $\sim .01M$, but its entropy huge, $\sim M^2$.

The argument above has been criticized on the grounds that for the asymptotic observer, the time that passes between collapse and evaporation, is large, of order M^3 . Small quantum corrections of order $\frac{1}{M}$ could build up over this period to (perhaps) “fuzz out” the horizon, and falsify our conclusion that the information thrown into the black hole is causally separated from the external world on the time slice 99. It is for this reason that we have concentrated on the state of the interior. The proper time interval that has elapsed since collapse is of order one on this part of the time slice. Of course, the time slice 99 that we have chosen is, in this case, a rather strange one. It hugs the light cone for much of its extent.

Note that in two dimensions, and thus for near extremal charged black holes for which much of the region of formation and evaporation is two dimensional, the black hole’s temperature is independent of its mass, and the evaporation time is of order M . This removes some of the above objections. An even more rigorous justification of this argument can be found in the large N analysis of two dimensional black holes that we will present in section 4. There, the quantum corrections are of order $\frac{1}{N}$ while the time for evaporation of a black hole whose mass scales like N is of order 1. It seems to me that the only way to avoid the conclusion of this argument is to give up the assumption that the semiclassical approximation to quantum field theory correctly describes physics in regions of spacetime where the curvature is small. Precisely such a retreat from the conventional wisdom is proposed by the critics of this argument, though there seems to be no agreement among them as to the correct replacement for semiclassical quantum field theory. I think it is fair to say that the above argument establishes that the “subtle correlation” approach is far from conservative. It implies a radical rethinking of our approach to the quantum dynamics of spacetime.

The only one of these radical approaches that the author of the present lectures can make even a pretense of understanding, is that of Susskind[8]. Susskind’s criticism of the above argument goes right to the heart of quantum field theory. Indeed, in making the above argument we accepted without comment the assumption that the Hilbert space

of the whole system is a tensor product of states inside the horizon and states outside. This is the fundamental principle that allows us to conclude that information cannot be radiated to the outside on the basis of the examination of the states inside the horizon. This assumption appears to be valid in quantum field theory and in naive cutoff schemes like lattice theories of gravity. Susskind makes the point that it is in no way valid in string theory. If we cut space into two pieces, there will always be strings which straddle the boundary. It is to these “stranded” string states that Susskind looks for the origin of black hole entropy and the resolution of the paradoxes of Hawking evaporation. We will present a brief description of his work in section 6.

2.2. *The Remains of the Data*

Let us for the moment accept the argument given above, and examine its consequences. The fact that the external world has large entropy on the time slice 99 is already in contradiction with the lump of coal analogy. When a lump of coal has radiated most of the energy it absorbed from the laser beam, and its glow begins to fade, it has also radiated away most of the information in the Encyclopedia. We can go further however, and argue that the situation on the time slice 99 implies the existence of a large set of degenerate long lived remnant states of the evaporated black hole. Suppose, in accord with the subtle correlation hypothesis, that the information trapped in the black hole on the time slice 99 is later radiated into the external space³ over a time T . It is then contained in correlations between the late time radiation and Hawking radiation which was emitted before 99. The former is contained in a sphere of radius T centered at the black hole⁴ while the latter is mostly causally disconnected from this sphere. The Hilbert space of the external world is , to a good approximation a tensor product of the space of the late time radiation and that

³ This implies that our theory contains no globally conserved quantities carried solely by massive particles.

⁴ For simplicity we are neglecting the horizon radius of the black hole of mass $.01M$.

emitted before 99^5 . The state of the system is thus

$$|\Psi\rangle = \sum \phi_{na} |a\rangle |n\rangle \quad (2.1)$$

where $|a\rangle$ is a state of the early time and $|n\rangle$ a state of the late time radiation. The entropy of the state of the early time radiation:

$$S = - \sum |\phi_{na}|^2 \ln |\phi_{na}|^2 \quad (2.2)$$

is supposed to be large, of order M^2 . Now consider the integral of the energy density over a sphere of radius larger than T , but small enough to be spacelike relative to the region where most of the early time Hawking radiation is. This is, to good approximation, a positive operator which acts only on the states $|n\rangle^6$. Its expectation value is

$$\langle E \rangle = \sum |\phi_{na}|^2 E_n < .01M \quad (2.3)$$

The only way to make (2.3) consistent with (2.2) is to have many low lying energy levels E_n . The late time radiation must therefore occupy a large volume and the duration T of the last stage of Hawking evaporation must be very long. Thus, even if the information which fell behind the horizon of the black hole is eventually returned to contact with the external observer, it must be stored for a long time in a quasistable “remnant”. If we assume that the remnant entropy is of order M^2 , then the remnant lifetime implied by the above argument is of order M^4 which for large black holes is longer than the Hawking evaporation time, and typically longer than the age of the universe.

The foregoing argument suggests that unless one envisions major violations of causality (or of the semiclassical treatment of gravity, which may be the same thing) in regions of spacetime where the semiclassical approximation shows no evidence of breaking down⁷, one

⁵ Here again we invoke the tensor product structure of the Hilbert space in local quantum field theory.

⁶ If there is a hole in this argument, which I doubt, this is the most likely place for it to be. Radiation of large amounts of negative energy density might change our conclusions.

⁷ Again I remind the reader that 't Hooft and others have argued that there is evidence for large corrections to the semiclassical calculation, related to the fact that particles emitted as Hawking radiation have large Schwarzschild frequencies near the horizon. I do not understand these arguments well enough to give a coherent account of them, but this is very likely a function of my stupidity rather than an intrinsic deficiency of the arguments.

must accept the existence of quasistable remnants. Thus even if black hole evaporation eventually returns all the information about the collapsed system to the external observer, the actual process by which this occurs is very different from the cooling of a lump of coal. The “unitary S-matrix”, “subtle correlation” scenario reduces, for all practical purposes, to the remnant scenario.

Further evidence for this contention comes from the investigation of Hawking radiation from moving mirrors by Carlitz and Willey[9]. A moving mirror with a fixed trajectory (or indeed any fixed spacetime geometry coupled to a quantum system but not satisfying self consistent equations with the expectation value of quantum stress as a source) cannot satisfy exact energy conservation. However, Carlitz and Willey find that if the mirror is decelerated slowly enough to cause negligible violations of energy conservation, then the information that would have been lost if the deceleration had not occurred can only be returned over a very long time interval. A similar situation would obtain if we artificially reversed the collapse of the interior of the Schwarzschild geometry, sometime after the spacelike surface Σ . If we did this in a way that approximately preserves energy conservation, we would find that the geometry could only return to flat space and the information return to the external observer, over a very long period. This is particularly clear in the synchronous gauge description of black hole geometry which we described in the introduction. If we wait a long time before reversing the collapse, the spatial geometry contains a large “horn” (see Fig. 1). In order to shrink this horn and return to a flat spatial geometry without significant violation of energy conservation, we require a long time.

I have emphasized Susskind’s string theoretic criticism of the basic tensor product structure of the space of states in quantum field theory vitiates the force of these arguments. What I find hard to believe is that these criticisms significantly affect the description of the evolution of a large black hole inside its horizon, long before the singularity is encountered. Thus I believe that the picture of long hornlike geometries connected on to our space by tiny holes must be valid in string theory as well as in ordinary quantum field theory. As we will see, consideration of such structures leads one to the notion of stable remnants with infinite numbers of degenerate states, even if one ignores the above arguments. Rather than

being in contradiction with the existence of remnants, I believe that Susskind's description of black hole evaporation is like the membrane picture of classical black hole physics: if it succeeds it will provide a complete description of Hawking radiation and Bekenstein-Hawking entropy from the point of view of the external observer. It does not contradict the alternate description of physics as seen by an infalling observer. In the final section we will make some comments about the differences between these two points of view. Their reconciliation relies on the distinction between information theoretic and thermodynamic entropy in situations in which different parts of the universe become causally separated from each other.

We turn now to an exposition of the apparent phenomenological catastrophe caused by the hypothesis of stable or quasi stable black hole remnants. Hawking's calculation of the evolution of an evaporating black hole appears to be valid until the mass of the hole is of order the Planck mass. Its Schwarzschild radius is then of order the Planck length, and it appears pointlike to all but the most well equipped external observers. The remnant scenario thus appears to require the existence of a new class of "particles" all of which have masses of order the Planck mass. On the other hand, there must be a distinct remnant particle, for each possible state of matter that can collapse to form a black hole, and since remnants can be formed starting with black holes of arbitrarily large mass, there must be an essentially infinite number of different remnant species. Even a few species of stable Planck mass particles might cause difficulties for cosmology if they are produced with reasonable probability after inflation, but an infinite number of species is a complete disaster. Schwinger's calculation of the pair production of charged particles in a background electric field show that the probability depends only on the mass and charge of the particle (for fixed field strength). An infinite number of degenerate charged species would give an infinite cross section for this process. (Similarly, the formula for Hawking production of a particle around a fixed black hole depends only on its mass.) We apparently stand in danger of producing an infinite shower of black hole remnants every time we turn on the lights. One of the most important ideas discussed in this review will be the demonstration that these estimates of remnant production are completely wrong, because black hole remnants do not behave like elementary particles even though they look

pointlike to an external observer. This argument will be taken up in the next section.

2.3. Information Loss

First however we must review the third major scenario for the endpoint of Hawking evaporation, that proposed by Hawking himself. Eschewing both the "information is returned in subtle correlations" approach and the idea of remnants (essentially for the reasons outlined above) Hawking instead proposed that an evaporating black hole simply disappeared, taking with it the information that was lost to the external observer in the collapse process. For very small black holes the entire process of formation and evaporation occupies a small region of space time. Since it is (in Hawking's view) a completely local phenomenon, it should happen all the time in the form of virtual processes even when sufficient energy for real black hole formation is unavailable. Again due to the locality of the process, we should be able to construct a coarse grained, or effective, theory, describing the effect of these virtual information destroying processes on large scale physics. Hawking indeed proposed a formalism for computing such corrections. Since pure states can now evolve into mixed states the effective theory must now map density matrices to density matrices in a way which does not preserve purity.

In ordinary quantum mechanics the initial and final density matrices in a scattering experiment are related by

$$(\rho_{out})_A^B = S_A^C (\rho_{in})_C^D S_D^\dagger{}^B \quad (2.4)$$

where S is the scattering matrix. Hawking proposed instead a general linear relation⁸

$$(\rho_{out})_A^B = (\$)_{AC}^{BD} (\rho_{in})_D^C \quad (2.5)$$

If the $\$$ matrix factorizes into the product of an S matrix and its inverse, then we have unitary evolution, preserving purity. If it does not so factorize then purity is lost. Hawking

⁸ Nonlinear density matrix evolution equations lead to nonlocal phenomena which Polchinski[10] has dubbed "Everett Phones". EPR correlations can be used to send messages in such theories. It is not clear to exactly what extent this is ruled out by experiment, but we will not discuss nonlinear density matrix evolution in this review.

proposed[11] that the true $\$$ matrix of the world had a small, nonfactorizable term of the form

$$\delta(\$)_{AB}^{CD} = C_{ij} \left(\frac{1}{M_P^p} \int O_i \int O_j \right) \quad (2.6)$$

where the O_i are operators of high dimension, as indicated by the powers of the Planck mass.

Unfortunately these apparently small corrections to the $\$$ matrix are not small at all. To see this[12] we will have to make a small extension of Hawking's $\$$ matrix formalism and discuss local time evolution equations for the density matrix. That this must be possible follows from the assumption that processes of virtual formation and evaporation of small black holes are, in Hawking's picture, confined to a small space time region. With sufficient coarse graining, we must be able to incorporate their effect in a set of local evolution equations. We will see that even if we take the coarse graining scale to be a nuclear time scale, the "small" terms in the evolution equation analogous to (2.6) are far from negligible. Let us begin by writing the most general linear coarse grained evolution equation for the density matrix:

$$\dot{\rho}_A^B = \mathcal{H}_{AC}^{BD} \rho_D^C \quad (2.7)$$

We now impose the conditions that ρ remain hermitian and that probability be conserved. To do this it is convenient to expand the four index object \mathcal{H} as a sum of tensor products of matrices. We then find the most general probability and hermiticity conserving linear equation to be[13]

$$\dot{\rho} = i[H, \rho] + C_{\alpha\beta} [O_\alpha, [O_\beta, \rho]] \quad (2.8)$$

where H is hermitian and $C_{\alpha\beta}$ is a real matrix. The O_α run over a complete set of hermitian operators. To preserve the positivity of ρ we must impose a condition on the relative sizes of the symmetric and antisymmetric parts of C . We *might* also want to impose a condition guaranteeing that entropy always increases [12].

When C is a symmetric matrix, it is possible to make a simple model which produces the equation (2.8). It is simply quantum mechanics coupled to random sources via a

hamiltonian

$$H_R = H + J_\alpha(t)O_\alpha \quad (2.9)$$

where the J 's have white noise correlation functions

$$\langle J_\alpha(t)J_\beta(s) \rangle = C_{\alpha\beta}\delta(t-s) \quad (2.10)$$

This interpretation makes most of the important features of the equation (2.8) obvious. In particular, although the evolution equation is time translation invariant, it does not conserve energy. Time translation invariance guarantees only the conservation of the average energy. In a random system there will generically be energy fluctuations and the moments of the energy will not be preserved. Similarly, space translation invariance of (2.9) does not guarantee conservation of momentum.

The extent of this violation of the conservation laws depends on the extent to which the operators O_α and the correlation function of the sources are local. We have assumed that we are working at a scale for which the time correlation of the sources is local. Hawking's proposals lead one to expect all the nonlocality in the new terms in the equation to be at the Planck scale. As shown in [12] this leads to disaster. The inverse powers of the Planck length are cancelled by matrix elements of the local operators between states of very low and very high energy. In a flash, the vacuum is converted into a mixed state whose dominant components have very high energy. To make the violations of purity small we have to smear the operators over long distance scales, which leads to violations of locality.

Special examples exist in which the non quantum mechanical terms in the evolution equation involve only conserved charges. However, all such examples violate either the positivity of the density matrix, or Lorentz invariance [14]. It is also unclear how such nonlocal equations for the density matrix could arise from integrating out processes which are localized in space and time.

It has been argued by Hawking[15], that these arguments must be wrong. Adopting Dyson's [16] idea that information loss is due to the splitting off of a disconnected universe, Hawking argues that such a process must conserve energy and momentum. This argument is correct, but overly classical. In a generally covariant quantum theory, the amplitude for splitting off a disconnected universe must be summed coherently over all points of

spacetime. Both the time and place where the splitting occurs, are infinitely uncertain. The result of integrating out such processes, is an effective action which is infinitely nonlocal in time as well as space. In the theory of wormholes it is shown that the sum over topologies can be rewritten as a theory in which the constants of nature are uncertain, but once they are determined by experiment, the local dynamics of the universe is described by quantum mechanics [17]. Splitting off of disconnected universes does not lead to evolution equations for the density matrix of the kind studied in [12], and cannot describe the spatially and temporally localized process of black hole formation and evaporation.

I should not conclude this section without mentioning a possible loophole in the argument. There is nothing that prevents us from writing a nonquantum mechanical evolution equation for the density matrix that has an extremely small dimensionless parameter in front of the double commutator term. The coefficient is severely constrained by the long lifetimes of metastable nuclei that alpha decay via quantum tunneling processes, but no experiment can constrain a coefficient to be exactly zero. A theory of information loss which naturally explained the small dimensionless parameter, would be consistent with experiment.

An interesting model which attempts to incorporate this idea has been constructed recently by Strominger[18]. He modifies the RST[19] model of two dimensional black hole evaporation by treating the model as an open string theory with a string coupling that depends on the values of world sheet fields in an unusual manner⁹. The string splitting and joining interaction vanishes unless the world sheet dilaton field takes on the critical value corresponding to world sheet black hole formation. In the semiclassical, large N limit, this only occurs near the worldsheet position of black hole formation. The probability of its occurring anywhere else is suppressed by a factor e^{-N} . This model then appears to be an example of true information loss that occurs with high probability only near the location of a classical black hole. The virtual processes envisioned by Hawking and shown to be incompatible with experiment in [12], are suppressed by a small dimensionless parameter.

Susskind has argued that this model does not produce world sheet physics compatible

⁹ That is, he considers open string theory with a singular spacetime dilaton (not to be confused with the world sheet dilaton) field.

with the existence of local observers, and in particular, that it violates the world sheet cluster property. His argument can be phrased as follows. Following the rules of string theory, the amplitude for two events of “black hole evaporation/ baby universe emission” is given by an integral

$$\int dx dy T(A(x)B(y)) \quad (2.11)$$

where both integrals are taken over the entire world sheet. This is a very nonlocal expression from the world sheet point of view. In the “conventional” approach to wormhole physics, the superposition of amplitudes with an arbitrary number of universe creation and annihilation events is shown to be equivalent¹⁰ to a superposition of amplitudes for local field theories with different values of the couplings. Each of these separately satisfies cluster decomposition.

Strominger instead proposes to enforce approximate world sheet locality by insisting that world sheet physics is approximately classical and that the vertex operators are nonvanishing only in the vicinity of certain localized classical events. However, the time ordering in (2.11) appears to foil this attempt. Consider two sequential black hole formation processes separated by a large proper time in the classical geometry. Cluster decomposition requires that the \$ matrix for this joint event approximately factorize into the \$ matrices for the single black holes. The time ordering in the string theory formula requires us to write the joint amplitude as a superposition of two amplitudes corresponding to emission of the “first” baby universe from either the “first” or “second” (according to the classical evolution) black hole. This enforces correlations on the joint dollar matrix which are incompatible with factorization. This ingenious attempt to avoid the [12] constraints on information loss, appears to fail.¹¹

¹⁰ It is assumed that the baby universes are always very small at the time of their connection onto the rest of the world, and the equivalence referred to above applies only to physics at a length scale larger than the typical baby universe size.

¹¹ M.Srednicki (*Private Communication*) has pointed out that in the absence of actual experiments on evaporation black holes we have no right to invoke the empirical success of the postulate of cluster decomposition. Note also that Polchinski and Strominger[20] have recently presented an extension of Strominger’s work and attempted to answer Susskind’s objections. I have been unable to understand their paper, but perhaps one should not conclude prematurely that the idea

3. Horned Particles as the Endpoint of Hawking Evaporation

The first intimations of a way out of the problems of the remnant scenario appeared in an unpublished note by Dyson, very soon after Hawking's seminal papers on the evaporation of black holes. The interior geometry of a black hole has, as we have seen, the structure of a collapsing horn or neck. In the case of a black hole formed in collapse, the collapsing matter sits down at the very end of the horn. Dyson envisaged a "pinching off" of the horn, which would leave the collapsed matter in a separate closed universe, thus "explaining" the loss of information found by Hawking.

In fact, as noted above, the "pinching off" of a closed universe cannot be adequately described by the classical picture of Dyson. A closed universe carries vanishing energy and momentum, and by the uncertainty principle, the event of "pinching off" can be localized neither in space nor in time. Dyson's idea leads instead to what Sidney Coleman has dubbed "spacetime wormholes"¹².

Although Dyson's picture is not directly relevant to the black hole information problem, his idea that the information "just goes somewhere else" turns out to be crucial. Chronologically, the second step in the unravelling of the mystery came with the work of Guth and Farhi [21] on a subject they facetiously called "the creation of a universe in the laboratory". In modern versions of the inflationary universe, our current horizon volume has inflated from a very small part of a much larger universe. It is not necessary for that larger space to have undergone inflation in order to create the universe that we see. Guth and Farhi asked whether a similar event could occur somewhere in our own universe. In other words, could a small region of the universe find itself in a false vacuum, and if so, what would occur? If one doesn't think too hard about this problem it seems to pose a paradox. The false vacuum region has a higher energy density than the true vacuum and so the interface between them is subject to an inward pressure. Our flat space instincts tell us that the false vacuum region should shrink. On the other hand, it has a positive cosmological constant, and therefore it should grow. Of course, dynamical curved space is able to accommodate both of these features. The false vacuum region evolves into an

of information loss to wormholes is dead.

¹² Not to be confused with the spatial wormholes of Wheeler.

“embolism” (Fig. 5), a large inflating DeSitter space connected onto ordinary Minkowski space by a rapidly shrinking neck.

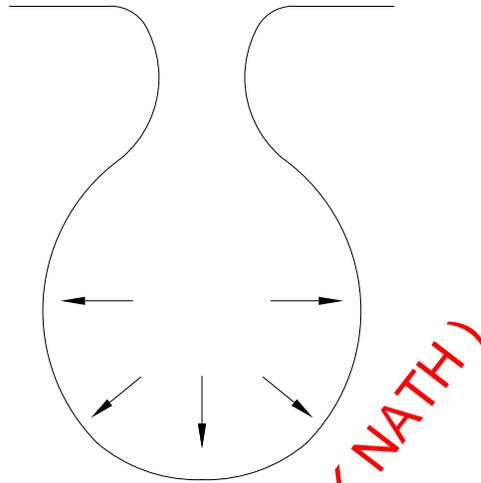


Fig. 5: The Guth-Farhi Embolism: An Entire Universe Looks Like a Black Hole Remnant to An External Observer

By Birkhoff’s theorem the geometry outside the neck looks like a black hole and we have an example of a black hole with a large internal geometry¹³. Subsequent evolution is generally said to lead to “pinching off” of the neck, but all this really means is that the neck evolves to a Schwarzschild singularity where the equations of motion break down and we cannot predict what really occurs without the aid of a more elaborate theory of the short distance structure of spacetime.

Suppose that the collapse of the neck is stabilized by some effect in this mythical short distance theory. The resulting object will look pointlike to a flat space observer with limited resolving power. He might be tempted to call it a particle. However, having been present at the creation, Guth and Farhi tell us that it actually contains an entire universe, perhaps one as large and as rich as our own. Surely one must suspect that its behavior might be quite different from that of an ordinary elementary particle, particularly when

¹³ Guth and Farhi showed that the initial configuration with a localized region of false vacuum could not evolve classically from nonsingular initial conditions, but argued that it might arise from quantum tunneling

it comes to questions about whether it can be created from the vacuum by the action of weak external fields. This then is a type of geometrical object that seems quite reasonable from the point of view of general relativity, especially if we are willing to invoke (as yet ill understood) quantum mechanical or string theoretic mechanisms to avoid the occurrence of singularities. The basic premise of these lectures is that objects like the Guth-Farhi universe, are not just a particular way to make a black hole, but are the natural endpoint of evolution for all black holes.

We now come to the important question of a name for almost pointlike (from the point of view of an external observer) black holes with large internal geometries. In the context of the physics of near extremal charged black holes, where this circle of ideas arose, I proposed that they be called *horned particles*, or *cornucopions*, “to celebrate both the shape of their internal geometry and the wealth of information hidden inside them”. The name is, I think, appropriate for this general class of object, because, as described in the introduction, the Schwarzschild geometry itself is a horned particle. To see this, we must dare to look behind the horizon. The celebrated Schwarzschild singularity is (in internal Killing coordinates), nothing but the squeezing down of the internal world into an infinitely long, infinitely thin horn. The “horny” aspects of the internal geometry of a black hole are quite generic. Even if we suppose that the transverse squeezing is stabilized at some very small radius, there is no reason to suppose that the stretching stops as well. Indeed, we will see in the next section that models of near extremal charged black hole collapse and evaporation, lead to internal spaces which have large extent only in one dimension.

4. Near Extremal Charged Black Holes

The Hawking temperature of Reissner-Nordstrom black holes in Einstein-Maxwell gravity, vanishes in the extremal limit $Q = M$. This is easily understood in terms of the geometry of the extremal black hole. For $Q = M$ the Killing vector which is timelike at infinity is everywhere timelike, the singularity is a timelike curve, and there is no horizon at any finite distance. Although quantum field theory on this background requires some kind of boundary condition on the singularity, it has a time independent Hamiltonian.

Quantum fields propagating on the background are in pure states¹⁴.

It has therefore seemed plausible to many researchers that extremal charged black holes are the endpoint of Hawking evaporation for the case where a black hole manages to retain its charge. It must be emphasized that this may be a very rare occurrence in the real world. For a charged black hole, Schwinger pair production acts to drain off the charge with the Hawking radiation. Given the known spectrum of light charged particles it seems implausible that a black hole could retain a very large electrical charge. Magnetically charged black holes may have a better chance of retaining their charge because magnetic monopoles are undoubtedly very heavy. However, in modern theories of elementary particle physics, monopoles arise as classical solutions to the field equations, which may have internal variations on scales much larger than the Planck length. The Reissner Nordstrom solution for pointlike monopoles, might have little to do with these objects[22] unless the magnetic charge is very large. In that case the scale of the Reissner Nordstrom geometry is much larger than that of the internal structure of the monopole, and the magnetic field at the horizon is very weak. We might hope that such highly charged objects are quasi-stable. The large (magnetic) charge limit is also the limit in which the geometry becomes smoothly varying and we can hope to avoid the necessity of understanding short distance physics in our discussion of Hawking evaporation.

Most researchers have decided to ignore the possible real world difficulties with the idea of stable charged black holes. The charged, near extremal black hole is viewed as a model in which the phenomenon of horizon formation and evaporation can be studied within the framework of a controlled approximation scheme. One hopes to extract lessons from it that may be applicable to more realistic but less tractable problems and in particular to neutral black holes¹⁵. We will follow this tradition, and say no more about the unrealistic features of charged black holes.

¹⁴ I am ignoring problems caused by the Cauchy horizon. There is a region of the spacetime, including the whole asymptotic region, which is causally separated from the Cauchy horizon.

¹⁵ Recent work[23] on exact string theoretic classical solutions for the region of space inside the horn of a dilaton black hole seems to indicate the existence of extremal *neutral* black holes, with transverse sizes of the order of the Planck length.

The Reissner-Nordstrom solution is not lightlike geodesically complete. Much of the initial work in this area concentrated instead on the geodesically complete charged black hole solution of the version of scalar-tensor (dilaton) gravity that arises in string theory[24]. Although this solution is also singular, it is geodesically complete and the singularity is at infinity. More importantly, in the appropriate conformal frame, the singularity can be attributed to the blowup of the parameter which controls quantum fluctuations, which suggests that quantum effects may well cure the singularity. Indeed, portions of the geometry, shown in Fig. 1, are very similar to the geometry encountered in 1 + 1 dimension soluble string theories[25] . This geometry, whose form was the origin of the name cornucopion, has a large region in which spacetime is approximately two dimensional. The solution [24]is, in that region, identical to the classical background of 1 + 1 string theory, in the limit of vanishing tachyon condensate μ . In string theory, Knizhnik Polyakov Zamolodchikov (KPZ)[26] scaling tells us that the effective loop expansion parameter is $\frac{1}{\mu}$. The perturbation series for the S matrix can be exactly computed and various resummations of it have been proposed[27]. All of them have the property that the $\mu = 0$ *quantum* theory is well defined, despite the blowup of the terms in the semiclassical expansion. One may therefore hope that the singularity of the GMGHS solution is similarly cured by quantum mechanics.

We emphasize again however that the whole focus of the work on charged black holes is to find mechanisms that resolve the Hawking paradox. One should not put too much emphasis on a particular model. Indeed, after the breakthrough in understanding the dilaton gravity models, Strominger and Trivedi reanalyzed the ordinary Reissner-Nordstrom black hole and argued that similar results about the existence of infinite numbers of degenerate states and effective information loss could be established in that context as well, by concentrating on regions far from the singularity. Indeed this model may be argued to avoid the strong coupling problems of the GMGHS solution.

One final piece of justification before proceeding to an analysis of the GMGHS solution: in a realistic quantum string theory, we expect the dilaton to acquire a mass, drastically changing the form of the black hole solution in the asymptotic region. However, unless the dilaton effective potential blows up in the horn of the cornucopion, it will not change the

physics of the two dimensional region on which we shall concentrate[28]

4.1. The CGHS Lagrangian

In theories containing both a metric and scalar fields, the definition of the metric is ambiguous because of the freedom to perform Brans-Dicke transformations. These scalar field dependent Weyl transformations can change the structure of the lagrangian and the apparent form of solutions to the field equations, but cannot change the result of any physical measurement. It is however often convenient to work in some particular Brans-Dicke frame.

In string theory, the obvious choice of conformal frame is that dictated by the string loop expansion. This is computed in terms of a two dimensional conformal field theory whose lagrangian has the form

$$\mathcal{L}_{worldsheet} = G_{\mu\nu}(x)\partial x^\mu \bar{\partial} x^\nu + \dots \quad (4.1)$$

The metric $G_{\mu\nu}$ which appears in this lagrangian is known as the *string* metric or σ *model* metric. At energies low compared to the string scale, the lagrangian for the string metric, the electromagnetic field and the dilaton takes the form:

$$\mathcal{L} = e^{-2\Phi} \sqrt{-G} (R + 4(\nabla\Phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}) \quad (4.2)$$

Garfinkle Horowitz and Strominger[24] found simple classical solutions of this lagrangian which are conformal transforms of the magnetically charged dilaton black holes of Gibbons and Maeda[24]. In the case of extremal charge, the solution takes the form

$$ds^2 = -dt^2 + e^{4\Phi} dx^2 \quad (4.3)$$

$$e^{2\Phi} = e^{2\Phi_0} + \frac{2Qe^{\Phi_0}}{|x|} \quad (4.4)$$

$$F_{\mu\nu} dx^\mu dx^\nu = Q \sin\theta d\theta d\phi \quad (4.5)$$

Notice that the geometry, depicted in Fig. 1, is completely nonsingular and has an infinite region which is approximately a cylinder with a two sphere cross section. The radius of the two sphere is of order Q . For large Q the curvature is everywhere small. The only

singular aspect of the solution is the blowup of the string coupling as we move down the horn towards $x = 0$.

Deep inside the horn, it is convenient to make a "Kaluza-Klein" expansion of the fields in spherical harmonics. It is easy to see that, apart from the usual Kaluza Klein $SO(3)$ gauge field, all of the non spherically symmetric modes propagate as massive waves inside the horn. Thus, there is an effective two dimensional theory of long wavelength modes propagating in the horn[29]

$$\mathcal{L}_{CGHS} = e^{-2\Phi} \sqrt{-G} (R + 4(\nabla\Phi)^2 + \frac{1}{Q^2} - \frac{1}{4}F^2 - \frac{1}{4}(G^a)^2) \quad (4.6)$$

Here all of the fields are two dimensional, F is the two dimensional electromagnetic field strength, and G^a is the field strength of the $SO(3)$ gauge field which serves as the carrier of information about three dimensional spatial rotations in the effective field theory.

In the two dimensional region, the extremal CGHS solution has the simple form

$$G_{\mu\nu} = \eta_{\mu\nu} \quad (4.7)$$

$$\Phi = \frac{x}{Q} \quad (4.8)$$

which is called the linear dilaton vacuum. The lagrangian (4.6) also has solutions representing nonextremal black holes which have the form

$$G_{\mu\nu} = \eta_{\mu\nu} e^{2\Phi} = (1 - Mx^+x^-) \quad (4.9)$$

Here, M is the deviation of the four dimensional black hole mass from extremality. In the literature it is often called the mass of the two dimensional black hole. There are solutions corresponding to dual charged and rotating black holes as well.

Dilaton black holes with large magnetic charge are certainly classical solutions of heterotic string theory. In that theory, or in any other containing light electrically charged fermion fields, these solutions will have a large number of fermion zero modes. Far away from the position of the black hole, these will be the Callan-Rubakov [30] modes of charged fermions around a magnetic monopole. They fall into angular momentum multiplets with angular momentum $J = Q - 1$. (In most of the literature, they are mistakenly called

S-wave modes. They are only S-wave for minimal Dirac magnetic charge.) Deep inside the horn of the black hole, these modes propagate as massless two dimensional fermion fields coupled to the gauge fields. The effective lagrangian is

$$\bar{\psi}_N(i\hat{\partial} - e\delta_{NM} + J_{NM}^a \hat{G}^a)\psi_M \quad (4.10)$$

In the existing literature, the couplings to the two dimensional electromagnetic and Kaluza Klein gauge fields have been neglected. The fermions were treated as free apart from their gravitational interactions, and were bosonized into a set of free scalar fields, giving the famous lagrangian of Callan, Giddings, Harvey and Strominger (CGHS)[31]:

$$\mathcal{L} = \sqrt{|g|}(e^{-2\phi}[R + 4(\nabla\phi)^2 + 4\lambda^2] - \frac{1}{2}[(\nabla f)^2 + \dots]) \quad (4.11)$$

In fact, one can reproduce the results of CGHS in a way that justifies the neglect of the gauge fields.

The free fermion lagrangian is a conformal field theory. It contains operators, $\bar{\psi}\gamma^\mu(1 \pm \gamma_3)\psi$ and $\bar{\psi}\gamma^\mu(1 \pm \gamma_3)J^a\psi$ which satisfy a $U(1) \times U(1) \times SU(2) \times SU(2)$ Kac-Moody algebra, where the nonabelian part has level $2Q - 1$. The full stress tensor of the theory can be written

$$T_{fermion} = T_{Sugawara} + T_{singlet} \quad (4.12)$$

where $T_{singlet}$ commutes with all the Kac-Moody currents. It has central charge which is proportional to Q for large Q . The fundamental operators out of which the singlet stress tensor is constructed, are bleached parafermions, defined by multiplying the original fermions by appropriate path ordered exponentials of the Kac-Moody currents. They are not local with respect to the fermion fields, nor even with respect to all of the fermion bilinears. Nonetheless, the operator $T_{singlet}$ and the fermions are relatively local. Thus we can imagine constructing by local operations a state of the system in which $T_{singlet}$ has a classical expectation value, but the Kac Moody currents all have expectation value zero¹⁶. Then, no gauge fields will be excited, only a two dimensional gravitational field. The resulting classical equations in the gauge $g_{\mu\nu} = \frac{1}{2}e^{2\rho}\eta_{\mu\nu}$:

$$0 = \frac{\delta S}{\delta \Phi} = e^{-2\Phi}(4\partial_+\partial_-\rho + 8\partial_+\Phi\partial_-\Phi - 8\partial_+\partial_-\Phi + 2\lambda^2 e^{2\rho}) \quad (4.13)$$

¹⁶ What is not at all clear is how to do this starting from the full four dimensional theory

$$0 = \frac{\delta S}{\delta \rho} = 2e^{-2\Phi}(2\partial_+\partial_-\Phi - 4\partial_+\Phi\partial_-\Phi - \lambda^2 e^{2\rho}) \quad (4.14)$$

$$0 = \frac{\delta S}{\delta g^{++}} = T_{++\text{singlet}} + e^{-2\Phi}(4\partial_+\Phi\partial_+\rho - 2\partial_+^2\Phi) \quad (4.15)$$

(with a similar equation for $\frac{\delta S}{\delta g^{--}}$) are identical with those of CGHS. They have solutions corresponding to the creation of a black hole by an infalling pulse of leftmoving radiation. However, the quantum expectation value of the trace of the stress tensor T_{singlet} is large, of order Q , in any region where the curvature becomes of order 1. Thus one is led to construct a $\frac{1}{Q}$ expansion in which this quantum correction, but no other, is taken into account self consistently. A systematic expansion is possible when the black hole mass is of order Q in the large Q limit. The resulting CGHS equations are

$$0 = e^{-2\Phi}(4\partial_+\partial_-\rho + 8\partial_+\Phi\partial_-\Phi - 8\partial_+\partial_-\Phi + 2\lambda^2 e^{2\rho}) \quad (4.16)$$

$$0 = 2e^{-2\Phi}(2\partial_+\partial_-\Phi - 4\partial_+\Phi\partial_-\Phi - \lambda^2 e^{2\rho}) - 2\kappa\partial_+\partial_-\rho \quad (4.17)$$

$$0 = T_{++\text{singlet}} + e^{-2\Phi}(4\partial_+\Phi\partial_+\rho - 2\partial_+^2\Phi) - \kappa((\partial_+\rho)^2 - \partial_+^2\rho + t_+(x^+)) \quad (4.18)$$

(and $++ \rightarrow --$), where the functions t_{\pm} are determined by the boundary conditions. $\kappa = e^{2\Phi_0}Q$, is the usual rescaled coupling constant. The nature of the solutions of these equations has been discussed extensively in the literature [32]. Here I will recall only a few salient points of that analysis. The solutions do exhibit Hawking radiation, as well as the backreaction of the geometry to that radiation. The apparent horizon (defined as the place where $\partial_+\phi = 0$) recedes from the observer. However, all of the solutions inevitably encounter a singularity. The reason for this is apparent if we note that (4.17) is singular when $e^{-2\Phi} = \kappa$. Essentially this is because, thinking of the system as a nonlinear model with Φ and ρ parametrizing the target space, the target space metric has zero determinant at this point. Writing $\Phi = -\frac{1}{2} \ln \kappa + \Delta$ and taking linear combinations of the trace and dilaton equations near $\Delta = 0$ we obtain

$$(e^{2\Delta} - 2)\partial_+\partial_-\rho = -2\partial_+\partial_-\Delta \quad (4.19)$$

$$(e^{2\Delta} - 1)\partial_+\partial_-\rho = \frac{3}{2}(4\partial_+\Delta\partial_-\Delta + \lambda^2 e^{2\rho}) \quad (4.20)$$

Since $e^{-2\rho}\partial_+\partial_-\rho \propto R$, the scalar curvature, we will have a curvature singularity unless the right hand side of (4.20) vanishes when $\Delta = 0$. However, the expression on the right hand side involves only first derivatives of the fields and will not vanish for generic initial conditions. In particular, if the initial state of the system is the linear dilaton vacuum, in which the field Φ attains the value $-\frac{1}{2}l n \kappa$ at a finite point, we should expect generic perturbations of it to produce a singularity. The Penrose diagram for these singular solutions resembles that of an ordinary black hole.

In attempting to resolve the singularity of the CGHS equations one might imagine that it were necessary to resort to the full machinery of short distance quantum gravity or string theory. However, it is not clear that this is the case. What is going wrong here is that the coupling $e^{2\Phi}$, is getting large. Our effective two dimensional field theory was derived from string theory in the classical approximation. Perhaps the strong coupling corrections to the effective lagrangian are large, but the degrees of freedom that we have isolated remain classical, and the evolution of the black hole can still be studied in terms of them. We will explore this possibility in the next section.

4.2. Non Singular Lagrangians

If we imagine integrating out the string theoretic and four dimensional degrees of freedom that are neglected in the CGHS lagrangian, we should expect to get a general renormalizable lagrangian for the graviton, dilaton, gauge fields and fermions. The lagrangians that we will study are not the most general possible lagrangians for these fields. First of all, as before, we will not discuss the gauge interactions. Furthermore, we will assume that the singlet conformal field theory with stress tensor $T_{singlet}$ is unchanged by higher order corrections. There is no real justification for this assumption, we make it only to keep the system “soluble”. Up until this point our considerations could be justified (for large Q) as a systematic approximation to a realistic model. This will no longer be the case. We are now studying a model of black hole formation and evaporation which may not correspond to anything in the real world (even the real world according to string theory). Nonetheless, our model is a close cousin of a realistic one, and its general features could well persist in the more complicated systems that we are unable to analyze.

The most general renormalizable lagrangian for the graviton dilaton system is

$$\mathcal{L} = \sqrt{-g}(D(\Phi)R + F(\Phi)(\nabla\Phi)^2 - V(\Phi)) \quad (4.21)$$

In the weak coupling limit $\Phi \rightarrow -\infty$, we have

$$D \rightarrow e^{-2\Phi} \quad (4.22)$$

$$F \rightarrow 4e^{-2\Phi} \quad (4.23)$$

$$V \rightarrow \frac{1}{Q^2}e^{-2\Phi} \quad (4.24)$$

The kinetic term in the lagrangian will be nonsingular for all Φ if $D'(\Phi) \neq 0$. If this is satisfied, the only singularities can come from the region of field space $\Phi \rightarrow \infty$. However, it is easily verified from the explicit solutions which we will present below, that if

$$D \rightarrow e^{n\Phi} \quad (4.25)$$

$$W \rightarrow e^{m\Phi} \quad (4.26)$$

with $n \geq m - 2$ as $\Phi \rightarrow \infty$, then the solutions are nonsingular there as well[33]. Here, W is an auxiliary function which we will define below.

To solve these models it is convenient to do a field dependent Weyl transformation $g_{\mu\nu} \rightarrow e^{2\Sigma(\Phi)}g_{\mu\nu}$, which eliminates the F term. We obtain a lagrangian

$$\mathcal{L} = \sqrt{-g}(D(\Phi)R + W(\Phi)) \quad (4.27)$$

(with $W = e^{2\Sigma}V$). Furthermore, since $D'(\Phi) \neq 0$, we might as well take D to be the independent scalar field in this lagrangian. We will use the notation $W(\Phi(D)) \equiv W(D)$. Note that the equation of motion for D involves no derivatives, so that this can be viewed purely as a gravitational lagrangian with higher curvature interactions. It is a general result that all solutions of all lagrangians of this type have a Killing vector, and that D is constant along the Killing flows[34]. Asymptotically, we want to tie on to the black hole solutions of the CGHS lagrangian and we thus take the Killing vector to be timelike at infinity. For this simple system, Killing horizons are precisely event horizons. In Killing coordinates the event horizons are thus at points where $\frac{dD}{ds} = 0$.

Our system is two dimensional, and we can thus find a Killing coordinate system which is also conformal $g_{\mu\nu} = \eta_{\mu\nu} e^{2\sigma(s)}$. In this gauge a complete system of equations for the metric and dilaton is

$$\frac{d^2 D}{ds^2} = 2 \frac{d\sigma}{ds} \frac{dD}{ds} \quad (4.28)$$

$$\frac{d^2 D}{ds^2} = 2e^{2\sigma} W \quad (4.29)$$

Using the freedom to shift the conformal factor by a constant, the general solution is given by quadratures as

$$e^{2\sigma} = \frac{dD}{ds} = \int^D W(x) dx \quad (4.30)$$

The single physical integration constant is the constant of integration in the integral of (4.30). This integration constant plays the role of the ADM mass. It is important to emphasize that the linear dilaton vacuum is not a solution of these equations. This is as it should be. The equations we have written are supposed to reflect the effective result of integrating out short distance quantum fluctuations. We should only expect their solutions to resemble the classical solutions of the theory in the extreme weak coupling region. It is not even correct to identify the ADM mass as the coefficient of the leading correction to the linear dilaton behavior at infinity. What we should do instead is to find a truly static (global timelike Killing vector) solution of the equations and define the ADM mass to be extremal at that value of the integration constant for which the static solution is realized. Nonextremal ADM mass corresponds to deviation from this static solution.

The nature of the solutions clearly depends crucially on the behavior of the function $U(D) \equiv \int_{D_0}^D W(x) dx + M$ where D_0 is chosen so that $M = 0$ is the static solution (if any). If $U(D)$ has precisely one real isolated zero for all values of M , the geometry has a single horizon, which is always a finite spacelike distance away from finite points in the external region. To see this, note that the distance is given by

$$distance = \int ds e^\sigma = \int \frac{dx}{\sqrt{U(x)}} \quad (4.31)$$

There is no natural endpoint for Hawking radiation in such models. We may expect that when coupled to matter fields they will lead to runaway solutions which Hawking radiate

forever [35]. In [33] models with two, generically isolated, zeroes were studied. These have solutions with the causal structure of Reissner-Nordstrom black holes, but without the timelike singularities of the Reissner-Nordstrom solution. In particular, there is an extremal value for M at which the two zeroes coincide. In this case, the horizon recedes to infinite spacelike distance, as one can see from (4.31). The resulting metric is truly static, and quantum fields placed in it will not Hawking radiate. However, it is not lightlike geodesically complete, and like the extremal RN solution, it has a Cauchy horizon.

Martin O’Loughlin has recently invented another class of lagrangian, which has a more satisfactory candidate for a remnant solution. In [33] no fine tuning of parameters was allowed. The idea was to find generic behaviors of large classes of lagrangians. In this way it was hoped that one could mitigate the fact that we do not know how to calculate the functions D, F and V in realistic examples. O’Loughlin’s analysis applies to a large class of lagrangians but requires one fine tuning of coefficients. Returning to the case where $U(D)$ has only one real zero for all M , he tunes a parameter in the lagrangian so that for $M = 0$ (by convention), a pair of complex zeroes merges with the real zero, making it into a triple zero. Then, the spacelike geodesic distance to the horizon becomes infinite in the extremal limit, and with the proper asymptotic behavior of $W(D)$ as $D \rightarrow 0$, we can make the interior of the black hole asymptotically DeSitter. The Penrose diagram of the black holes in this model is that of Fig. 2. The distance to the horizon is finite in the nonextremal case and infinite for $M = 0$.

In order to study Hawking evaporation of these nonsingular black holes, we must couple them to matter and compute the backreaction in some kind of large N approximation. When the back reaction term is added to the equations, we must impose a stronger condition $|D'(\Phi)| > \kappa$, where κ is the rescaled large N coupling, in order to avoid a CGHS type singularity. With this constraint imposed, the equations were studied numerically for the case of $U(D)$ with two zeroes, by O’Loughlin and Lowe [36]. As mentioned, this case is very similar to the Reissner-Nordstrom geometry and [36] also carried out a numerical study of evaporation of Reissner-Nordstrom black holes. This case had been treated previously by Strominger and Trivedi [37], using approximate analytical techniques. The evolution is

completely nonsingular¹⁷, and leads to the following qualitative picture (Fig. 6).

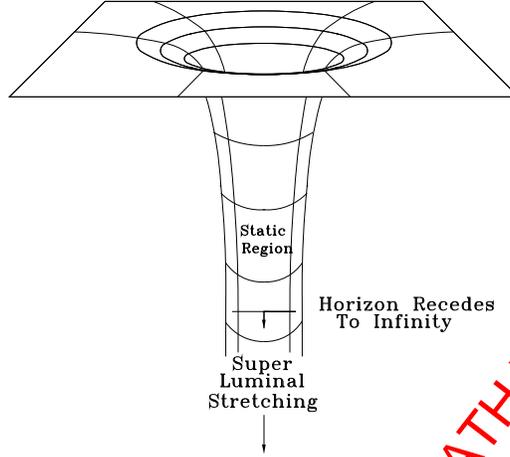


Fig. 6: Evolution of the Spatial Geometry of a Nonextremal Black Hole Back to an Extremal Remnant

When matter is incident on an extremal black hole, an apparent horizon is formed and the black hole begins to radiate. The apparent horizon recedes from the external observer, eventually leaving an infinite static geometry identical to the exterior of the extremal black hole solution. The full spacetime however has a horizon, and one can verify explicitly that the state of the field theory behind the horizon depends on the nature of the initial infalling matter. Thus, within these models, we have a consistent remnant scenario for black hole evaporation, in which information is lost to the external observer. There is a different type of remnant for each kind of initial pulse that forms the black hole (and thus an infinite number of different remnants altogether), but they are all indistinguishable from the point of view of the external observer. The information that distinguishes between them is causally disconnected from him. On any finite time slice, the system still has an apparent horizon that is accessible to the external observer, and it may be possible for him to associate the information with states on the horizon. Asymptotically he cannot do so. The apparent horizon has gone off to infinity, a different infinity than his own initial

¹⁷ There is of course a timelike singularity in the Reissner-Nordstrom case, but this does not effect the qualitative nature of the evolution

asymptotic region. His S-matrix will not be unitary.

One issue that has not been sorted out by these studies is the fate of the Cauchy horizon of the initial black hole. More properly, one should ask whether the true spacetime geometry, the solution of the large N equations, has a Cauchy horizon. All that is known for sure from the numerical work is that no singularity was encountered in the interior of the horizon within the time span covered by the calculation. Possible causal structures that could be attained via nonsingular evolution, which would not contain Cauchy horizons, have the form of Fig. 7 (Fig. 2 is a special case of this).

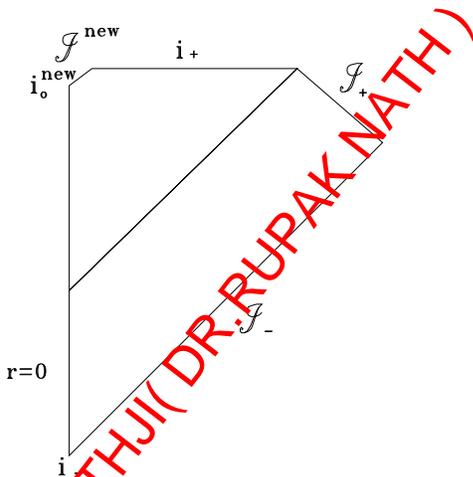


Fig. 7: Possible Causal Structures of Black Hole Remnants

That is to say, the structure would be similar to that of the classical solution of O’ Loughlin’s triple zero model. It would be of interest to resolve this issue and/or to study the large N evolution of O’ Loughlin’s model. With regard to the latter project, one should take note of the probability that once quantum corrections are taken into account, we will have to retune the parameters of the lagrangian to have a horizon that is an infinite distance away.

4.3. Production of Remnants from the Vacuum

I now come to the discussion of the famous “problem” of the remnant scenario, the spectre of infinite production cross sections. In doing so I must confess to a gross error that

I have made in much of my published work on this subject, which has misled others as well as myself. In early work[38] I stated that the infinite numbers of degenerate states of the cornucopion were in fact causally connected to the external world. They were supposed to be stationary quantum excitations of the static geometry located far down the horn of the cornucopion. Their degeneracy was attributed to the fact that they were infinitely far away. In fact, as we have seen, the picture is quite different. On any finite time slice in a synchronous gauge, a cornucopion is *not* a static geometry. It has an apparent horizon, behind which the spatial geometry is undergoing superluminal expansion. The infinite number of degenerate states of the cornucopion are the states of quantum fields lying behind this horizon.

The model for all discussions of production of nontrivial classical field configurations from the vacuum of a quantum field theory, is a classic paper of Affleck and Manton[39]. These authors studied pair production of 't Hooft-Polyakov monopoles in a constant magnetic field, and showed that in the small field limit the amplitude reduced to Schwinger's formula for the corresponding amplitude for elementary monopoles. The essential idea of the calculation is that for sufficiently small accelerations, the only degree of freedom of a soliton that one should be able to excite is the noncompact collective coordinate for its motion through space. Thus, for the motion of a soliton in a weak external field, one can construct an approximate solution in which the soliton simply follows a curved world line. The idea of Affleck and Manton was that one can compute the pair production cross section for solitons in the same field by analytically continuing the solution for a pair of particles to Euclidean time. The uniformly accelerated hyperbolic Minkowski motion continues to a circle in Euclidean time and one computes the production amplitude as the exponential of the action of this Euclidean instanton.

We do not really understand the Euclidean continuation of quantum gravity, but at the semiclassical level, there is a long tradition of seemingly sensible calculations[40] which simply take over the idea of analytically continuing Minkowski geometries. Special provisions must be made for situations in which the vector field which we use to define time at infinity, changes signature in some finite portion of the spacetime. The analytic continuation is performed by taking this vector field to be imaginary, but this only produces

an Euclidean manifold in the region where the vector field is timelike. On the surface on which the change takes place the vector field is null. In the analytically continued space-time, this null surface is replaced by a point, and the portion of the Minkowski manifold beyond this null surface is discarded. Now consider the pair production of the degenerate states of cornucopions. Unlike solitons, these are not globally static configurations. They have, to a good approximation, a timelike Killing vector outside the apparent horizon¹⁸, but it becomes null on the apparent horizon. Following the rules of Euclidean gravity as we know them, the Euclidean continuation of a cornucopion trajectory does not contain the portion of space where the degenerate states of the cornucopion live. Thus, the tunneling process cannot produce these states.

Although this is the basic argument against infinite production cross sections advanced in [41], that paper actually addressed a slightly different question, and was motivated by the mistaken idea that the degenerate states were causally connected to the external world. The issue was whether the infinite static geometry of the cornucopion could be produced from the vacuum by weak external fields. As we have seen, this is not really the correct question to ask if we are worried about the degenerate remnant issue, but it is nonetheless an interesting question. The intuitive idea behind the argument of [41] was that unlike a t Hooft Polyakov monopole, the internal structure of a cornucopion was unstable to small perturbations. Classically, this is related to the Second Law of Black Hole dynamics. Any small perturbation of the extremal black hole should produce a horizon. In [41] it was argued that this was true in the case of perturbations that tried to move the center of mass of the cornucopion. If this is so, then analytic continuation of the moving solution will lead to an instanton which creates only finite spatial volumes, namely only the volume of space in front of the horizon of the moving black hole. This had been shown previously for extremal Reissner Nordstrom black holes by Garfinkle and Strominger[42]. Recently, this claim has been challenged in the case of dilaton black holes in string theory [43], but it is unclear whether the infinite instantons that have been exhibited really contribute to production processes. The quantum corrections around them are not well defined. One should also be wary of studying the process in a truly constant background field. The

¹⁸ This approximation becomes better and better as time goes on and the horizon recedes.

resulting spacetimes are not asymptotically flat and it may be that one can get confused about what states are really being produced. This issue deserves further study, because it may have a bearing on the single aspect of this entire subject that might be amenable to observational verification, the discovery of cornucopions.

4.4. Remnants of the Imagination?

If black hole remnants exist, will we ever be able to find them? And if we do, will we ever be able to tell what they are? Unfortunately, the answer to the first question is probably no. Cornucopions are by hypothesis stable. Although we have no very good idea about the processes which might produce them in the very early universe, it is unlikely that the production process is so finely tuned that it can produce a density of remnants that is neither much larger nor much smaller than the density of ordinary matter in the universe. In the former case, cornucopions would be ruled out by observational astronomy, in the latter we would never be able to find them. We can save a model of the production process that produces too many cornucopions by invoking inflation, but in that case there are likely to be no black hole remnants in our portion of the universe. We would have to wait for macroscopic black holes to finish their Hawking evaporation before we could get our hands on real remnants.

But suppose we did so. Would we then be able to tell what the remnants were by their properties, or would they just behave like ordinary elementary particles? Classically I believe the answer to this question is no. Small perturbations of the cornucopion change its internal structure and produce horizons¹⁹. If we now imagine quantizing the classical cornucopion solution, it would formally have statistics, like any other elementary particle. However, any realistic experiment in which we attempted to measure these statistics would be doomed to failure. The procedure of scattering the cornucopions to measure their statistics would invariably cause the formation of an internal horizon and the emission of Hawking radiation that would change the internal state in an uncontrollable way. The statistical phase would be unmeasurable, and the cornucopions would behave like classical

¹⁹ Though this is where the issues raised by [43] must be faced.

distinguishable particles. A double slit experiment for cornucopions would resemble Fig. 8.

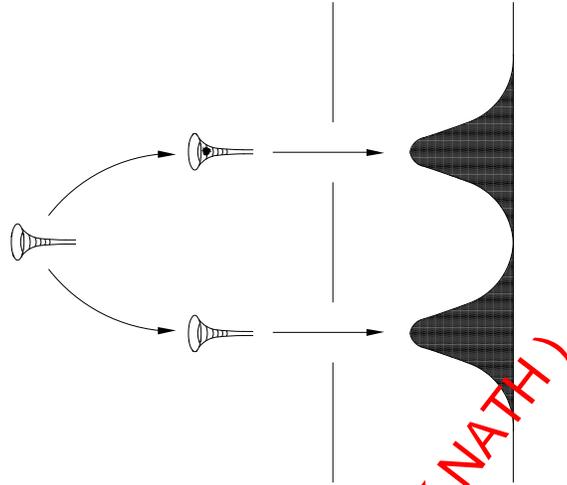


Fig. 8: A Double Slit Experiment for Cornucopions

Apart from the objections of [43], there is another way in which the above argument could fail. Quantum mechanically there might be a threshold energy for horizon formation, as there is in the two dimensional model of Russo, Susskind and Thorlacius. Given their infinite internal geometry, it is inevitable that the spectrum of excitations of cornucopions will have a continuum, but the continuum might be separated from the ground state by a gap. Experiments in which the cornucopions were moved with sufficiently small velocity would not excite the continuum and in these experiments they would behave like quantum particles. Determination of the size of the gap in the cornucopion excitation spectrum seems to me to be one of the most interesting open problems in the study of black hole remnants. Unfortunately, it is a very hard problem, and probably requires the study of strongly coupled string theory.

Finally one should note that the pessimistic assessment of the probability of finding black hole remnants in the first paragraph of this section might be wrong. Perhaps the differences between them and elementary particles are sufficiently great that their production probability in an inflationary universe is much larger than we presently imagine.

5. Unitarity, Information Loss and CPT

The general picture that arises from the discussion of the previous section is that black hole collapse and evaporation leads to the formation of a new asymptotic region of space which is causally disconnected from the old asymptotic region. The spacetimes that we have discussed can be foliated by spacelike hypersurfaces and within the semiclassical approximation for the geometry, there will be no breakdown of unitarity. Local physics will continue to obey the rules of quantum mechanics.

It is clear however that the S-matrix measured by observers in the original asymptotic region will not be unitary. The cornucopion spacetime has a horizon buried deep within the horn of the cornucopion. On any spacelike slice, the state of the system outside the horizon, which is all that can ever be probed by the original asymptotic observer, is correlated with that inside the horizon. The external density matrix is not pure, and the purely external scattering process must be described by a Hawking ρ matrix.

The cornucopion scenario thus unifies the idea of information loss in the observable universe with the idea of black hole remnants, retaining the merits of both while discarding their difficulties. In particular, Hawking's original claim that formation and evaporation of "small virtual black holes" would lead to a reformulation of the fundamental microphysical laws to accommodate time dependent information loss is discarded, and with it the problems unveiled in [12]. Virtual quantum fluctuations of geometry that do not change the topological properties of space, will be describable as distortions of the classical background over small volumes, which subside after a short time. No new asymptotic regions are created in such fluctuations, and it should be possible to "integrate them out" and construct a local effective lagrangian for long distance physics. This local Lagrangian will obey the usual rules of quantum mechanics. Small, topology changing processes can also be integrated out [17], and they too lead to a picture in which the information content of a single connected component of the universe does not change with time. Thus, to paraphrase J.A.Wheeler, we have *Information Loss Without Information Loss*. The rules of quantum mechanics are locally preserved but the global S-matrix will not be unitary unless we take into account contributions from causally disconnected asymptotic regions.

Hawking has emphasized that any remnant scenario for black hole evaporation implies

a violation of CPT in the observable universe. This is indeed the case. Suppose that it is possible to scatter a pair of elementary particles with sufficiently large energy and momentum transfer that they form a black hole. The resulting hole will Hawking radiate, and asymptotically settle down into a cornucopion state in which a new asymptotic region of space is formed. The CPT inverse of this process involves a conspiracy of the matter in this infinite asymptotic region, which causes the “internal universe” to collapse and spew out precisely two particles into the region of space in which we live. On the other side of the horizon in the meantime, particles must be sent in from infinity to converge on the point where the cornucopion throat is sitting, and be absorbed by it as inverse Hawking radiation. Such initial conditions, involving as they do a rather special conspiracy between causally separated points in two infinite universes, are clearly of measure zero in the space of all possible initial conditions. CPT is violated in the same way that it usually is in macroscopic systems: the inverses of processes which increase coarse grained entropy require very special initial conditions, which can never be realized in practice. Once cornucopions are formed, there is essentially zero probability that they will spontaneously dissipate. Similarly, there is zero probability that two cornucopions of opposite charge will annihilate when the mouths of their horns meet. Even if their charges are opposite, the probability that their internal states are exactly CPT conjugates of each other is zero. This is not just a question of the huge number of possible degenerate states available in the large interior of the cornucopion. The interior state of the cornucopion is time dependent, and involves an expanding geometry. The CPT conjugate state is one with a contracting geometry and very special initial conditions. It will not be realized in a cornucopion which formed from the collapse of matter in the external world.

Indeed the annihilation process for cornucopions should be the time reverse of the production process that we discussed above. That process, as we saw above, was both very improbable, and resulted in production of cornucopions of very small volume. Thus, in order to annihilate large volume, expanding, cornucopions, we must first force them to evolve classically into the configuration which is produced at the end of the tunneling process described in [41]. This is very unlikely to happen. Furthermore, causality prevents it from happening as a consequence of simply moving the mouths of the cornucopions so

that they coincide in the external space. Cornucopion production, whether through the improbable process of pair production in an external field followed by classical expansion of the internal geometry, or through gravitational collapse, is an effectively irreversible process from the point of view of a single causally connected sector of the universe.

In passing, we note that the above discussion is the strongest argument in favor of the existence of neutral black hole remnants. We can certainly imagine bringing together the mouths of two oppositely charged cornucopions, obtaining a neutral object. The above arguments suggest that that neutral object will have a large and complicated internal geometry, which will not disappear. It will be a neutral cornucopion. This is even more apparent if we imagine neutralizing the charge of a cornucopion by dropping an elementary particle of opposite charge into it. This microscopic perturbation cannot destroy the complicated internal geometry. It is perhaps not surprising then that Giddings, Polchinski, and Strominger have found what appears to be the infinite horn of a *neutral* cornucopion as an exact classical solution of string theory[23].

5.1. Entropy

No discussion of information loss and unitarity in black hole physics is complete without mention of the Bekenstein-Hawking entropy formula. It is common to describe this entropy as a measure of the number of internal states of a black hole. In the context of the cornucopion scenario this cannot be the case. In this picture, a black hole has an enormous internal geometry and an essentially infinite number of internal states.

A black hole of mass M could be in any one of an infinite number of states depending on its past history. The information theoretic entropy of the external world would depend on which state the hole was in. However, most of the information contained in the correlations between the internal and external states of the black hole can have no impact on the future interaction of the black hole with the external world, because it is causally disconnected from the external portion of spacetime. Thus, this measure of entropy can have no bearing on the *thermodynamic* entropy of the hole, which describes how it exchanges energy with the outside world, and by definition can depend only on our choice of spacelike surface in the exterior.

Indeed, imagine an experimenter on Earth preparing a large number of Einstein Rosen Podolsky pairs of neutrinos, and sending one particle of each pair off to a distant galaxy. The local state of the Earth is thus rendered highly impure and incoherent. However, the state left behind by the departing neutrinos does not resemble that produced by a black body sitting a finite distance from the earth. The process of pumping energy into the system of escaping neutrinos (by sending more neutrinos out) will not be describable by the laws of thermodynamics. The neutrinos do not interact with the Earth once they have been emitted. A system can come into thermal equilibrium only with a collection of states with which it is in causal contact, and constantly exchanging energy.

The thermodynamic entropy of a large black hole can only be related to states located very close to the horizon. If we look at states localized at any finite distance from the horizon, with a resolution coarse enough so that the semiclassical picture of the geometry is valid, then these states are expanding away from the horizon at superluminal velocities. They are out of causal contact with the outside world, and in the cornucopion scenario they will remain so forever. It is only states “infinitesimally close” to the horizon that can be in causal contact with the outside. Here infinitesimally close probably means within a length scale l which is small enough so that experiments probing physics on the scale l cannot be described by the semiclassical approximation. This probably means that l is of order the Planck length, or the slightly larger fundamental length of string theory. It is thus plausible that the number of states of a black hole that might be in causal contact with the external world is proportional to the area of the horizon, and it is perhaps natural that the proper units (i.e. those in which the proportionality constant is of order one) of area are Planck units²⁰.

The fact that the information theoretic entropy of a black hole must, in the remnant scenario, be thought of either as infinite (the logarithm of the number of possible final states of the remnant) or as dependent on the black hole’s entire previous history (the entropy of entanglement of the internal state produced in a particular process of formation of the hole with the external world) has been among the many arguments levelled against the remnant

²⁰ In string theory, the string length is the more natural unit. This discrepancy will have important consequences below.

scenario. I believe that the above paragraph shows clearly that neither of these quantities is a relevant measure of the thermodynamic entropy of the hole. Further it is clear that if the black hole has a thermodynamic entropy, it should be proportional to the area of its horizon. A microphysical demonstration that the vicinity of the horizon really contains the number of states indicated by the Bekenstein- Hawking entropy formula would seem to depend on knowledge of physics at very short distances. Here we make contact with the point of view of 't Hooft[44] who has long insisted that the divergence of the entropy of entanglement of the state of a quantum field theory outside the horizon with that of the inside was the key to understanding the BH entropy formula. 't Hooft believes that this demonstrates that the understanding of black hole entropy and of the information paradox is a problem of short distance physics and will guide us in the construction of the fundamental theory of small scale geometry. While I do not agree with his assertion that unitarity of the S-matrix for the original asymptotic region is a necessary ingredient in the construction of the theory, I do agree that the thermodynamic nature of black holes can only be understood in terms of short distance physics.

I have emphasized throughout this review that our discussion of the evaporation of black holes depends only minimally on Planck scale physics and not at all on quantum gravity. We have seen that it is possible to make the idea of information loss to an asymptotic observer consistent with the basic rules of unitary quantum field theory in a self consistently generated classical gravitational field. The microphysical derivation of the Bekenstein Hawking formula is the one aspect of this subject that still seems to hint at the need for a more fundamental theory. I believe that I have outlined a way in which this formula could be compatible with the idea of remnants, but the derivation of the formula itself seems to be outside the domain of reliability of the semiclassical field theoretic considerations that have been our guide up to this point. Furthermore, this discussion makes it clear that the remnant scenario (nor, I believe any scenario which leads to effective information loss) cannot account for the information corresponding to the BH entropy. In the next section I will describe one attempt that has been made to understand the Bekenstein Hawking entropy formula from a microscopic point of view.

6. Stranded Strings and Black Hole Entropy

The cornucopion scenario for black hole evaporation is a logically complete, (though technically incomplete) description of this process. By contrast, the material I am about to discuss is very much “work in progress”. Its authors (primarily ’t Hooft, Susskind, and their collaborators), do not pretend to have achieved a logically complete understanding of the process of Hawking evaporation. I am in the unfortunate situation of understanding even what these authors *have* accomplished only imperfectly. I enter into a public discussion of their work with great trepidation, and apologize in advance for my inevitable distortion of their points of view. All errors in the paragraphs that follow are my own.

One of the fundamental ingredients in ’t Hooft’s work on black holes is the Bekenstein-Hawking entropy formula. One starts from the assumption that black holes behave like thermal objects with Hawking temperature and Bekenstein-Hawking entropy, and attempts to make this fact consistent with the postulate that the S-matrix measured by the initial asymptotic observer is unitary. A special role for Schwarzschild coordinates is implicit in these assumptions. The black hole is static, and thermodynamic considerations make sense, only in these coordinates.

One of the most revealing of the calculations performed by ’t Hooft, is that of the entropy of states of quantum fields in a small region of a fixed Schwarzschild time slice, very near the horizon. ’t Hooft argues that in order for a quantum field theoretic calculation of the entropy and energy of the black hole to give an answer of the order of the classical values (i.e. M for the energy and the BH entropy), one must cutoff the quantum field theory by insisting that there are no states at a proper distance closer to the horizon than $\sqrt{\frac{N}{90\pi}} M_P^{-1}$ where N is the number of field degrees of freedom. But this removes many of the modes of the field which are involved in Hawking’s calculation of the outgoing density matrix. Modes which can escape to infinity with energies of order the Hawking temperature, have extremely high energy near the horizon and do not satisfy ’t Hooft’s brick wall boundary condition.

Susskind’s point of departure is the ’t Hooft entropy calculation described above. He brings in ideas from the Membrane paradigm, which has had many successful applications to the astrophysics of black holes. The basic idea of the membrane paradigm is that, from

the point of view of an external observer, things that fall through the horizon of a black hole may equally well be imagined to reside on a membrane suspended above the horizon. Susskind, Thorlacius and Uglum[2] proposed to take the membrane or “stretched horizon” to lie only a few fundamental lengths from the horizon (i.e the stretched horizon is very analogous to ’t Hooft’s brick wall), and postulated that the Hawking Bekenstein entropy can be understood in terms of degrees of freedom living on this membrane. They further suggested that the process of Hawking evaporation could be completely understood in terms of interactions between the external world and these degrees of freedom, and that the resulting S matrix would be unitary.

’t Hooft’s calculation shows that this suggestion makes no sense in quantum field theory. The number of Schwarzschild degrees of freedom per unit area of the membrane will be infinite in field theory, and will be much larger than $e^{\frac{M^2}{M_P^2}}$ for any cutoff that keeps all the degrees of freedom necessary to describe Hawking radiation. We have discussed previously an even more serious (since it does not make reference to very high energies or special coordinate systems) field theoretical argument that information cannot get out of a black hole with the Hawking radiation. This argument invoked the tensor product structure of field theoretic Hilbert spaces, causality, and the smoothness of the spacetime metric on a certain spacelike hypersurface. In field theory, if we tried to associate degrees of freedom with a stretched horizon, the Hilbert space would (by locality) be a tensor product of an inside space an outside space and a horizon space. We could make conclusions about information carried by the infalling particles by examining the inside space alone, and use the tensor product structure to make negative conclusions about the possibility of that information being carried by the horizon.

I would like to present two more arguments (due to Susskind) that the membrane picture cannot work in quantum field theory. To do so we consider a black hole of extremely large mass, or what is the same thing, Rindler space. The spacetime curvature is completely negligible for our considerations. We know that Rindler space is simply a wedge of ordinary Minkowski space. We want to consider observations made by a Rindler observer following a timelike trajectory whose proper distance from the Rindler horizon is a few multiples of the fundamental length.

Now let us consider throwing a wave packet of particles at the horizon. The wave packet is assumed to be localized in the transverse direction at infinity. How will such a packet appear to our Rindler observer hovering just above the horizon, according to the rules of quantum field theory? To some extent this is a difficult question because it involves scattering cross sections at very high energies and low momentum transfers. This regime is not well understood even in weakly coupled field theories. We are interested in the question of whether the cross section grows with energy, and in how much information is contained in the growing cross section. A rigorous answer to the question of growth of the cross section is not known, but what we do know fairly rigorously, both from field theoretic studies in models with spin one gauge bosons, and hadron phenomenology²¹ is that the growing part of the cross section is more or less universal. In parton model language, the quantum numbers that distinguish hadrons from one another are carried by the valence partons, which carry finite fractions of the longitudinal momentum of the hadron in the infinite momentum frame²². These behave like normal particles with transverse wave functions which do not grow with energy, but remain localized in the transverse plane. Their wave functions also Lorentz contract in a normal manner and are thin pancakes in the infinite momentum frame. The bulk of the high energy cross section is due to *wee partons*, which have a $\frac{dx}{x}$ distribution in longitudinal momentum and logarithmically spreading wave functions in the transverse plane. In hadron physics, the wee parton distribution is universal and contains no information about which hadrons are scattering (except perhaps whether they are baryons or mesons). We emphasize that the existence of growing cross sections and wee partons is a conjecture in field theory. If they do not exist then our argument is even simpler. The important point is that they do not carry (much) information even if they exist.

The implications of this behavior for our Rindler observer are clear²³. He will find that

²¹ Here we are extrapolating the behavior of hadrons at energies $\sim 1TeV$ to energies many orders of magnitude above the Planck scale

²² A rather appropriate frame for studying super-Planck energy collisions near the horizon

²³ Actually, the statements made above apply to an inertial observer who happens to have the same velocity as the Rindler observer at the moment he makes his measurements. If the measurements are sufficiently localized in space and in infinite momentum frame time there is

the the information in an infalling particle wave packet remains localized in the transverse plane. Furthermore if he restricts his attention to the region outside a stretched horizon which is some fixed distance from the horizon, he will soon find that he can not measure anything about the infalling state at all. The flat pancake which carries all the information falls below the stretched horizon in a time short compared to the Hawking evaporation time of the black hole. Thus, according to the rules of quantum field theory, it is impossible for a Rindler observer to imagine that the information carried by infalling particles gets smoothly spread over the stretched horizon , to be emitted later as isotropic Hawking radiation.

Susskind points out that in string theory the situation is quite different. Growing cross sections are built into string theory, since it is Regge behaved and the intercept of the graviton trajectory is 2. A very picturesque way of understanding this has been developed by Karliner Klebanov and Susskind[45] who did Monte Carlo simulations of the distribution of strings predicted by the wave functions of free bosonic string theory in light cone gauge. Consider a small box of fixed size in the transverse plane, and a string whose center of mass is in that box. KKS ask how much of the actual length of the string is in that box. The answer depends on a cutoff that they imposed on longitudinal momentum. Remembering that low longitudinal momentum means large light cone energy , we see that this is a cutoff on the time resolution of the observer looking at the string. Remember also that in light cone gauge string theory, a cutoff on longitudinal momentum is a spatial world sheet cutoff, a cutoff on the number of modes of the string. KKS find that as the cutoff is taken to infinity, the proportion of the string that is in the box goes to zero. This is a symptom of the logarithmic spread of the string in the transverse plane. The region (measured in string units) in which the string is confined grows logarithmically with the cutoff.

probably not very much difference between these two observers in quantum field theory. The distinction might be more important in string theory, where we do not really know how to solve the theory in a noninertial reference frame. One extremely important aspect of the situation that cannot be understood in terms of inertial frames is the difference between the experiences of the inertial and accelerated observer.

To apply this to the black hole problem, note that for a Schwarzschild observer supported near the horizon, watching a string fall into a black hole a time T (long) after the black hole is formed, the Lorentz boost between the observer's frame and the infalling string frame corresponds to a time dilation²⁴ $e^{\frac{TM_P^2}{4M}}$. Thus in order to see what is going on in the string frame the observer needs an exponentially fine time resolution, corresponding to a spread of the string over an area $\frac{TM_P^2}{4MM_S^2}$. Thus, in the Hawking evaporation time $T \sim \frac{M^3}{M_P^4}$ the string has spread over an area $\frac{M^2}{M_P^2 M_S^2}$. There is an important factor of the string coupling in these formulae corresponding to the fact that the natural length scale for string fluctuations is larger than the Planck length by a factor of $\frac{1}{g}$. This ensures that the string has spread over an area larger than the horizon, or rather that it has been able to cover the horizon many times. In higher dimensions the string spreading, evaporation time, and horizon area scale differently and the string has spread over regions bigger than the horizon by powers of the black hole mass²⁵

Another important difference between string theory and field theory is that the information which distinguishes between states is carried not by *valence partons*, which are localized excitations, but by vertex operators which are conformal fields smoothly spread over the fluctuating string. (Examples are the Kac-Moody currents which carry gauge quantum numbers.) The well known inability of string theory to reproduce localized form factors for hadrons is a symptom of this effect. This implies that the localized Schwarzschild observer will not be able to conclude that the information carried by an infalling string is

²⁴ Here we are treating the Schwarzschild frame as a highly boosted inertial frame, because we do not know how to do unitary string quantum mechanics in noninertial frames. This is a weak point in Susskind's arguments that deserves investigation.

²⁵ In all of these considerations, it is important to take the string coupling to be extremely weak. The region in which our description of string scattering is reliable is $\frac{g^2}{M_S^2} E < b$, the impact parameter. In order for the energy to be high enough for string spreading to cover the horizon, we need $\ln(\frac{E}{M_S}) = \frac{g^4 M^2}{M_S^2}$. However we also must be discussing experiments with impact parameters much less than the Schwarzschild radius of the black hole. Thus, we must have $\frac{M}{M_S} \gg 1$ and $g \ll \sqrt{\frac{M_S}{M}}$. This inequality is not satisfied by the string coupling in the real world. Thus Susskind's estimates are really valid only in an imaginary world with very weak coupling. I thank E. Martinec for a discussion of this point.

localized near its center of mass. A collection of observers spread over the horizon would be necessary to extract the information (and they would then probably completely change the state of the string). Thus there is no contradiction with the claim that the information can be emitted as isotropic thermal radiation.

In a similar vein, the Regge behavior of string scattering amplitudes and their growing cross sections, imply that strings do not Lorentz contract to sizes smaller than the string length. Thus, if we put a stretched horizon within a string length of the horizon, strings will never appear to “fall through it” from the point of view of a Schwarzschild observer. Thus in string theory, as opposed to quantum field theory, the picture of information carried by a stretched horizon which interacts with the outside world is not ruled out.

In assessing Susskind’s claims about string theory and black holes, it is important to understand that he is not proposing to change the picture of physics as seen by an infalling observer near a large mass black hole. The spacetime geometry is smooth and so is the observer’s coordinate system, and no string theoretic corrections to a semiclassical field theory description are important. This is not the case for the Schwarzschild observer. We have recalled that in classical general relativity the Schwarzschild observer can access all of the information about objects that long ago fell into the black hole, but only by making observations on extremely short distance and time scales. She can attribute all interactions of the black hole with the exterior to a membrane on the stretched horizon, but unless she takes her stretched horizon extremely close to the true horizon and measures time with extreme precision, she can only discuss average properties of the hole. It is to describe these extremely precise measurements of short times and distance scales that string theory, or some other theory of the small scale structure of the world, becomes necessary. Susskind’s string theory of black hole thermodynamics is, like the classical membrane paradigm, a description of the physics that is tied to a particular coordinate frame (and those related to it by smooth coordinate transformations).

We now come to what most researchers consider the most important argument against a unitary S-matrix, the analysis of the spacelike surface we called 99 in section 2. The refutation of this argument goes right to the heart of the difference between quantum field theory and string theory. The essence of the argument is that the state of the system on

this spacelike hypersurface lives in a Hilbert space which is a tensor product of an “inside” space and an “outside” space. If we wish, we can add a “membrane” space intermediate between the two, but this changes nothing. We claim that we know from the prior history of the system that the inside state is correlated with the outside. Then, from knowledge that the inside state has not changed very much (“nothing much happens to the infalling observer as he falls through the horizon of a large black hole”) we conclude that the outside density matrix has a large entropy. When we try to rerun this argument in string theory, we run into an immediate snag. Consider ordinary light cone gauge string field theory and let us try to divide space into two along one of the transverse directions. We want to consider the entire Hilbert space as a product of an $x_1 > 0$ and $x_1 < 0$ Hilbert space. We cannot. The full Hilbert space contains states which are created from the vacuum by creation operators for strings which straddle the boundary. These do not belong in either the plus or minus Hilbert space. The tensor product structure of the Hilbert space of any quantum field theory is completely unrecognizable here. For a black hole spacetime, these non field theoretic states will be strings “stranded on the stretched horizon”,

String field theory in a half space is in fact a complicated interacting theory. In addition to states of strings completely within the half space, it contains the stranded states described above. And even when the string coupling vanishes, the half space theory contains complicated interactions in which a closed string is annihilated and replaced by an arbitrary number of stranded strings. These are required to reproduce the dynamics of free strings moving in the full space, which cross the boundary (Fig. 9).

It is important to note that the wave functionals of even low energy states of the string have nonzero amplitudes for finding large strings in them. Thus the creation operator for a photon state with center of mass coordinate arbitrarily far from the boundary has a nonzero piece which creates stranded strings. It is only when we agree not to study the photon state on short time scales that we are able to ignore this. The large strings which appear with finite amplitude in the photon wavefunctional all have large world sheet wave numbers. When we consider averages over light cone time intervals much larger than the string scale we can construct an effective theory in which string degrees of freedom with large world sheet wave numbers are integrated out. The only large strings which appear

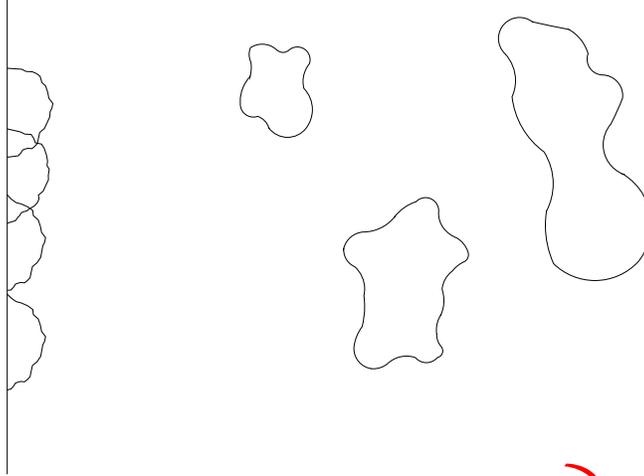


Fig. 9: Stranded Strings Violate the Locality Postulate of Field Theory, But Can Only Be Seen With Very Short Time Resolution

in this effective theory are large smooth strings. Neither the photon wavefunctional nor that of any low energy excitation of the string has any significant amplitude for such large smooth strings. Thus, the stranded string states in the full string field theory Hilbert space can be ignored if we are making low energy observations of low energy states.

The situation is quite different for observations with very short time resolution. Consider a wave packet of photons travelling in the negative x_1 direction, the center of which is at some large negative x_1 . The string wavefunctional for this photon has finite amplitudes for string configurations which wander arbitrarily far from the center of the wave packet, even into the region where $x_1 > 0$. These are configurations which contain large contributions from very high world sheet wave numbers, and they oscillate with very high frequency. However, measurements with sufficiently fine time resolution in the $x_1 > 0$ region should be able to discern them. In principle these measurements could determine the state of a photon whose center of mass was arbitrarily far away.

Susskind thus argues that much of the information (indeed he claims all the information) that in field theory is lost behind the horizon of the black hole, is encoded in string theory in the state of the stranded strings on the stretched horizon. It would be a remarkable consistency check of these ideas if one could calculate the entropy of stranded strings in a black hole background and find that it agreed with the Bekenstein Hawking

entropy. Unfortunately, things are not so simple. Firstly, we do not know how to formulate a Hamiltonian version of string theory in a black hole background, nor even in noninertial coordinates in Minkowski space. Secondly, even the problem of the entropy of a half space in inertial coordinates²⁶ is, in string theory, quite complicated. As noted above, it is an interacting string field theory of closed and open strings even when the string coupling is set to zero. Finally, there is a puzzle about the detailed form of the BH entropy formula in any theory in which the fundamental length is larger than the Planck length by a factor of the dimensionless coupling that controls semiclassical expansions.

In particular, in string theory, the fundamental unit of length is the string length $l_S \sim \sqrt{\alpha'}$, related to the Planck length by the dimensionless string coupling, $g_S l_S \sim l_P$. Thus, in terms of the natural units of string theory, the Bekenstein-Hawking entropy is an effect of order $\frac{1}{g_S^2}$. Any straightforward calculation of the quantum entropy of strings in a black hole background will, to leading order, be independent of g_S . Indeed, this would seem to be a property of any quantum theory. In the semiclassical expansion entropy arises by tracing over states which are small fluctuations about a classical background. To leading order, their description is independent of the coupling, and one would expect the expansion of the entropy to begin at order g^0 .

Susskind proposes to avoid this apparent difficulty by the following heuristic, though rather convincing, line of argument. Imagine doing string theory in a light cone frame moving parallel to the horizon. In this gauge we can, as above, try to make sense of local quantities in string theory. In particular, Susskind proposes to take over from local field theory the notion that an observer in this frame localized a distance d from the horizon, effectively sees a bath of strings at a d dependent temperature, approaching the Hagedorn temperature as $d \downarrow l_s$. If this is the case, then the string partition function at genus one will blow up in a black hole background. It has long been believed that in the finite temperature case, this divergence is the signal of a phase transition to a new phase of string theory. In particular, Atick and Witten[46] have argued that this phase transition is of first order. Formally, the precursor of the transition is a tachyonic mass for certain winding mode of

²⁶ This problem is of interest because in quantum field theory formal arguments indicate that the half space entropy is exactly equal to the Rindler entropy[5].

strings around the compact time dimension of a finite temperature cylinder. Atick and Witten argue that string interactions stabilize this instability, leading to a finite condensate of winding modes which contributes to the free energy at order $\frac{1}{g_s^2}$. Susskind proposes to identify the black hole analog of the genus zero entropy of Atick and Witten with the Bekenstein-Hawking entropy.

It is beyond the reach of present stringy technology to calculate the Atick-Witten entropy at genus zero, much less Susskind's profound generalization of it. Nonetheless, taken as a whole, Susskind's proposal is, in this author's opinion, the first serious attempt to explain the BH entropy in terms of a quantum mechanical sum over states. His whole picture, if viewed as the quantum mechanical analog of the Membrane Paradigm, is consistent with everything we know about black holes and strings. In particular, it does not contradict the use of effective field theory for the description of the vicinity of the horizon of a large black hole as viewed by an infalling observer. Classical general relativity tells us that a Schwarzschild observer with fine enough resolving power is able to retrieve all information about things that fell into the black hole in "the remote past"²⁷. It is only for the quantum mechanical analysis of these super-Planckian measurements that one needs to resort to string theory.

Viewed in this manner, the BH entropy formula hints at a very interesting conclusion. In classical physics, we can throw an infinite amount of information into a black hole, and the Schwarzschild observer can measure it. The BH formula suggests that this infinity is cutoff by quantum effects. Bekenstein has tried to argue [47] that this cutoff reflects a fundamental quantum gravitational restriction on the amount of information that can be contained inside a volume bounded by an area A . As far as I know, all such arguments implicitly assume that the volume enclosed is finite, and certainly do not take into account geometries with horizons which recede off to infinity. The cornucopion scenario is in direct contradiction with Bekenstein's bounds, if they are taken to refer to the entire entropy of entanglement of the external world with the world behind the black hole horizon. If instead, they are taken to represent only the entropy of states near the horizon that are

²⁷ as seen by an infalling observer. Of course all these measurements are made on a single Schwarzschild time slice.

in causal contact with the external world, the contradiction is removed. I believe that this is all that is necessary to prevent violations of the second law of thermodynamics in the presence of black holes.

Instead, in view of Susskind's picture of the origin of the BH formula, I would ascribe the finiteness of BH entropy to a limitation on the information carrying capacity of stranded strings on the stretched horizon. Consider a string state whose center of mass is thrown into the black hole. In free string theory it leaves behind stranded strings on the horizon. Once interactions are taken into account, these stranded strings can break, combine with other strings, and in this way, "lose contact" with the original state which deposited them. Information about the infalling state is now truly lost to the stranded strings and thus to the external observer. The horizon is thus a semipermeable membrane for information: that information represented by the BH entropy is information about the state of the stranded strings on the stretched horizon., but it is not complete information about what fell into the black hole. The idea that complete information is in principle accessible is an artifact of classical physics, corresponding to the divergence of the BH entropy in the $g_S \rightarrow 0$ limit.

This lecture has been only a brief summary of Susskind's ideas. I have stressed primarily those points where I felt that a clearer explanation than could be found in the literature was necessary. For more details, I refer the reader to Susskind's original papers.

7. Conclusions

I believe that by combining the notion of cornucopions with Susskind's ideas about string theory and BH entropy, we have for the first time the outline of a sensible story about the problem of information loss in black hole evaporation. These two seemingly contradictory aspects of the description are the analog of the infalling and supported observers' points of view in classical general relativity. The cornucopion scenario tells us what is "really going on" behind the horizon. It cannot easily account for the BH entropy, which is a notion relevant to the observations of the external observer²⁸. Susskind's

²⁸ Note that in the interpretation advanced in these lectures, the BH entropy cannot be identi-

interpretation of the BH entropy is a concrete realization of the heuristic treatment of this entropy by the membrane paradigm. It is remarkable that we have to give up the rules of quantum field theory in order to find a consistent realization of these semiclassical ideas. It is even more remarkable that string theory seems to provide the required generalization of field theory. If these arguments are verified, 't Hooft's bold claim that the resolution of the paradoxes of Hawking radiation would lead us to the correct theory of quantum gravity will be vindicated.

Taken by itself, Susskind's picture could be advanced (and Susskind so advances it) as proof that *all* of the information in the black hole is returned to the external observer. As indicated above, I do not think that this is necessarily correct. All of the recent work on remnants has led to a picture in which black hole formation and evaporation terminates, at least for black holes of sufficiently large magnetic charge, in a semiclassical spacetime in which the number of different causally disconnected asymptotic regions is different in the future than in the past. Basically, this is the picture of a black hole that comes out of classical general relativity, with only the singular behavior of that theory removed. I believe that perturbative string theory on such a spacetime would lead to the prediction that the S-matrix for the original asymptotic region is non-unitary (basically, because even though string theory is nonlocal, it satisfies the cluster property of the S-matrix).²⁹

It is somewhat of a jump, (and I think at odds with a lot of semiclassical reasoning about what happens to freely falling observers in regions of small spacetime curvature) , to conclude from the fact that string theory may explain the Bekenstein-Hawking entropy in terms of degrees of freedom connected with the horizon, that *no* information is lost in black hole evaporation. In the present state of our knowledge (and even accepting Susskind's claims about stranded strings as fact) one may equally well argue that the BH entropy formula simply represents a quantum string theoretic limit on the information carrying

fied with the entropy of entanglement of the interior and exterior of the black hole. The latter is an S-matrix concept, which can be defined in an observer independent manner. If our interpretation is correct, the thermodynamic BH entropy is a concept that is useful for the external observer only.

²⁹ Of course, the spacetimes in question are not solutions of the classical equations of string theory, so naive perturbative string theory is not quite applicable.

capacity of the degrees of freedom connected to the horizon. Perhaps this limit arises because (due to string interactions) the stranded strings that were originally connected to states whose center of mass fell through the horizon, can break, and lose contact with the information they originally carried. In this interpretation, the BH formula represents only that portion of the entropy of black hole formation that remains causally connected to the external observer. Such an interpretation seems consistent with the semiclassical description of the observations of an infalling observer who sees the formation of a new asymptotic region of spacetime.

On the other hand, someone who insists on a unitary S-matrix for the world outside a black hole must either deny the possibility of the formation of causally disconnected asymptotic regions of space or insist on a radical reformulation of our notions of spacetime and/or quantum mechanics. Such a point of view is certainly more exciting than the conservative approach that I have advocated. It leads to the notion of “holographic” theories of spacetime in which the number of states of a space with D dimensions is a finite multiple of the area of a transverse spatial slice like a black hole horizon [48]. There are even indications that string theory is such a holographic theory [49]. At the moment however, the intermediate position that I have outlined seems more plausible to me.

And this brings us back to the frustration with which I began this talk. Black hole evaporation is a purely theoretical subject. The only aspect of my discussion that is possibly accessible to experimental test (and that possibility is quite remote), is the claim that there exist pointlike composites that behave like macroscopic classical objects. Apart from that, the resolution of the controversy surrounding Hawking radiation appears to await the construction and qualitative solution of a mathematically consistent theory of quantum gravity. It is still possible for an honest researcher to uphold any of the three major points of view about the outcome of the controversy (though personally I feel that theories of true information loss are probably not consistent).

The avenue for future progress appears to branch into three rocky paths. The first is to pursue the study of remnants in string theory, and to try to derive from string theory the nonsingular models of remnants that have been constructed by hand. In particular, the question of whether there is a threshold for excitation of the continuum modes of

a cornucopion should be pursued vigorously, as well as the question of the number of cornucopions expected in our horizon volume. These questions bear on the only experimentally accessible aspect of our subject. The second path is the verification of the 't Hooft-Susskind picture of the states that are counted by the Bekenstein-Hawking entropy of a black hole[50]. I deliberately split off from this a third path: the study of holographic theories, that is theories in which the S-matrix of the external observer is exactly unitary. In particular one wants to ask of such theories how the unitarity of the external S-matrix of a black hole can be made consistent with the apparent feeling of an infalling observer that there are *asymptotic* states causally disconnected from the outside. Although string theory appears to exhibit nonlocality, it does seem to satisfy the usual rules of cluster decomposition of the S-matrix, and to preserve the solutions of the Einstein equations for large smooth geometries. For large mass black holes, there seems to be a clear indication of the existence of new asymptotic states, causally disconnected from the world outside the horizon. Indeed, as Susskind often emphasizes, we have no evidence that we are not living inside of a black hole³⁰. If string theory is holographic, and the external S-matrix unitary, it is very hard to understand how to interpret the experience of “internal” observers.

At present, all of these paths seem steep and rocky and difficult to climb. Those who follow them will surely want to console themselves with dreams of the fantastic vistas that will open up before them when they reach the top.

³⁰ In most inflationary models of the universe, this is certainly the case.

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References

- [1] D.A. MacDonald, R.H. Price, K.S. Thorne, **The Membrane Paradigm**, Yale University Press
- [2] L.Susskind, L.Thorlacius, R.Uglum, *Phys. Rev.***D48**,(1993),3743.
- [3] S.Hawking, *Comm. Math. Phys.***43**,(1975),199.
- [4] G. 't Hooft, *Nucl. Phys.***B256**, (1985),727.
- [5] M.Srednicki, *Phys. Rev. Lett.* **71**,(1993),666; B.DeWitt,*Phys. Rep.***19C**,(1975),297; W.Unruh, N.Weiss, *Phys. Rev.***D29**,(1984),1656; L.Bombelli, R.Kaul, R. Sorkin, *Phys. Rev.***D34**,(1986),373; D.Kabat,M.Strassler,*Phys. Lett.***B329**,(1994),46, hep-th 9401125; C.Callan,F.Wilczek, *Phys. Lett.***B333**,(1994),55,hep-th 9401072.
- [6] G. 't Hooft,*Nucl. Phys.***B335**,(1990),138.
- [7] E.Verlinde, H.Verlinde,*Nucl. Phys.***B406**, (1993),43.
- [8] L. Susskind, *Phys. Rev.***D50**,(1994),2700, hep-th 9309145; *ibid.***D49**,(1994),6606;*Phys. Rev. Lett.***71**,(1993),2367.
- [9] R.Carlitz, R.Willey, *Phys. Rev.***D36**,(1987), 2336; F.Wilczek, hep-th 9302096.
- [10] J.Polchinski, *Phys. Rev. Lett.* **66**,(1991),397.
- [11] S.Hawking, *Phys. Rev.***D14**,(1976),2460.
- [12] T. Banks, M.Peskin, L. Susskind, *Nucl. Phys.***B244**,(1984),135.
- [13] G.Lindblad, *Comm. Math. Phys.***48**,(1976),119.
- [14] M.Srednicki, *Nucl. Phys.***B410**,(1993), 143, hep-th 9206056.
- [15] S.Hawking, *Nucl. Phys.* **B244**,(1984),135.
- [16] F.Dyson, Institute for Advanced Study preprint, 1976, *Unpublished*.
- [17] S.Coleman, *Nucl. Phys.***B307**,(1988),864; S.Giddings, A.Strominger, *Nucl. Phys.***307**, (1988),354; T. Banks, *Nucl. Phys.***B309**,(1988),643, *Physicalia***12**, (1990),19.
- [18] A.Strominger, hep-th 9405094.
- [19] J.Russo, L. Thorlacius, L.Susskind, *Phys. Rev.***D47**, (1993),3444.
- [20] A.Strominger, J.Polchinski, hep-th 9407008.
- [21] A.Guth, E. Farhi, *Phys. Lett.***B183**,(1987),149.
- [22] K.Lee, V.P.Nair, E.Weinberg, *Phys. Rev. Lett.***68**,(1992),1100, hep-th 912008.
- [23] S.Giddings, J.Polchinski, A.Strominger, *Phys. Rev.***D48**,(1993),5748.
- [24] G.Gibbons,R.Maeda,*Nucl. Phys.* **B298**,(1988),741; D.Garfinkle, G.Horowitz, A.Strom-
inger, *Phys. Rev.***D43**,(1991),3140.
- [25] See **The Large N Expansion in Quantum Field Theory and Statistical Physics**, E.Brezin, S.Wadia, Editors, World Scientific, Singapore, 1993, for a comprehensive collection of articles on one dimensional string theory.
- [26] D.Knizhnik, A.M.Polyakov, A.B. Zamolodchikov, *Mod. Phys. Lett.***A3**,(1988),819.
- [27] G.Moore, M.R.Plesser, S.Ramgoolam, *Nucl. Phys.* **B377**,(1992),143, hep-th 9111035.

- [28] J.Horne, G.Horowitz, *Nucl. Phys.***B399**,(1993),169, hep-th 9210012; R. Gregory, J.Harvey, *Phys. Rev.***D47**,(1993),2411, hep-th 9209070.
- [29] C.Callan, S.Giddings, J.Harvey, A.Strominger, *Phys. Rev.***D45**,(1992),1005; S.Giddings, A.Strominger hep-th 9202004.
- [30] C.Callan, *Phys. Rev.***D26**,(1982),2058; V.Rubakov, *Nucl. Phys.***B203**,(1982),311.
- [31] C.Callan, S.Giddings, J.Harvey, A.Strominger, *Phys. Rev.***D45**,(1992),1005.
- [32] T.Piran, A.Strominger, hep-th 9304148; S.Hawking, I.Stewart, hep-th 9207105; B.Birnie, S.Giddings, G.Horowitz, A.Strominger, *Phys. Rev.***D46**,(1992),638; J.Russo, L.Susskind, L.Thorlacius, *Phys. Lett.***B292**,(1992), 13; L.Susskind, L.Thorlacius, *Nucl. Phys.***B382**,(1992),123; D.Lowe, *Phys. Rev.***D47**,(1993),2446; T.Banks, A.Dabolkhar, M.Douglas, M. O'Loughlin, *Phys. Rev.***D45**,(1992),3607.
- [33] T.Banks, M.O'Loughlin, *Phys. Rev.***D48**,(1993),698.
- [34] T. Banks, M. O'Loughlin, *Nucl. Phys.***B362**,(1991),649.
- [35] A.Bilal, C.Callan, *Nucl. Phys.***B394**,(1993),7.; S.R.deAlwis, *Phys. Lett.***B289**,(1992),278, *Phys. Lett.* **B300**,(1993), 330; S.Giddings, A.Strominger, *Phys. Rev.***D46**,(1993),2454.
- [36] M.O'Loughlin, D.Lowe, *Phys. Rev.***D48**,(1993),3735.
- [37] A.Strominger, S.Trivedi, *Phys. Rev.***D48**,(1993),5778.
- [38] T.Banks, A.Dabolkhar, M.Douglas, M. O'Loughlin, *Phys. Rev.* **D45**,(1992),3607.
- [39] I.Affleck, N.Manton, *Nucl. Phys.* **B194**,(1982),38.
- [40] S.Hawking, G.Gibbons, *Phys. Rev.* **D15**,(1977),2752; D.Gross, M.Perry, L.Yaffe, *Phys. Rev.***D25**, (1982),330; S.Coleman, F.Delucia, *Phys. Rev.***D21**,(1980),2133.
- [41] T.Banks, M.O'Loughlin, A.Strominger, *Phys. Rev.***D47**, (1993),4476.
- [42] D.Garfinkle, A.Strominger, *Phys. Lett.***B256**,(1991),146.
- [43] F.Dowker, J.Gauntlett, S.Giddings, G.Horowitz, *Phys. Rev.* **D49**,(1994),2909,hep-th 9312172.
- [44] G. 't Hooft, *Nucl. Phys.***B256**,(1985),727.
- [45] M.Karliner, I.Klebanov, L.Susskind, *Int. J. Mod. Phys.***A3**,(1988),1981.
- [46] J.Atick, E.Witten, *Nucl. Phys.***B310**,(1988), 291.
- [47] J.Bekenstein, *Phys. Rev.***D7**,(1973),2333, *Phys. Rev.***D9**,(1974),3292, *Phys. Rev.***D49**,(1994),1912.
- [48] G. 't Hooft, gr-qc 9310006; L. Susskind, hep-th 9409089
- [49] L. Susskind, op. cit.; C. Thorn, *Reformulating String Theory With the $\frac{1}{N}$ Expansion*, Talk Given at the First International A.D.Sakharov Conference, Moscow, Russian Federation, May 1991.; S.Das and A. Jevicki, *Mod. Phys. Lett.* **A5**,(1990),1639.
- [50] A.Dabolkhar, hep-th 9409158.